

# **Algebra 1**

## **MADE EASY**

**Common Core Standards  
Edition**

**Written by:  
MaryAnn Casey**

**Topical Review Book Company**

# **Algebra 1 Made Easy Handbook Common Core Standards Edition**

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## Acknowledgements

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## Introduction

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**Algebra 1 Made Easy - Common Core Standards Edition** is written to coordinate with the Common Core State Standards (CCSS). The CCSS is a nationally recognized set of standards that will help our math students become “college and career ready.” This simply means they will be prepared to be successful in whatever they choose to do following graduation. As individual states adopt the CCSS, students and teachers across the country will be working hard to implement it. Instruction will be changed significantly in most classrooms leading to more experimentation and hands on learning. Students will be encouraged to “think outside the box” when analyzing problems and determining how to approach solving them. It will still be necessary for students to be able to use accepted mathematical processes to complete the analysis and solution. These methods are explained in this handbook. Algebra I Made Easy is designed to be a casual, student friendly handbook. It is a quick reference guide for students to use to obtain help with understanding which mathematical procedures and methods are acceptable and how to use them. This handbook contains the content covered by the CCSS, not the teaching techniques and learning styles that will be developed. When the student has analyzed the problem and chosen a desired method of approaching it, this handbook will help them “do the math” correctly. I wish everyone success with our implementation of the Common Core and hope this handbook helps you to reach that goal.

Sincerely,

MaryAnn Casey,  
B.S. Mathematics, M.S. Education

# ALGEBRA 1 MADE EASY

## Common Core Standards Edition

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**Read all problems carefully.  
It is easy to make a mistake in the setup.**

### WORD PROBLEMS USING VARIABLES

1. Represent the variables with letters.
2. Translate the words of the problem into algebraic phrases.
3. Solve for the variable.
4. Substitute the value of the variable in the “let” statement to find the other answers (when necessary).
5. Check your answer in the WORDS of the problem.
6. Make a conclusion - answer the question the problem asks - use words.  
Such as: “The number is 5.” *or* “The rectangle is 6 by 7.”
7. MAKE SURE your answer is correct for the domain (real numbers unless indicated otherwise). Sometimes an answer must be rejected.

### APPLICATIONS WITH 2 VARIABLES

Some problems require that variables are used to represent quantities in the equation that is created. When two variables are needed, two equations are created. In certain problems, two equations are solved separately, in others the two equations are solved as simultaneous equations or as a system of equations (both are solved together).

#### Examples

- ① Joe’s Taxi service charges \$3.50 as a flat travel fee. He charges an additional \$0.45 per mile,  $x$ . Write an equation that represents his earnings,  $y$ .

$$y = 0.45x + 3.5$$

- ② Samuel has a jewelry business where his clients create their own bracelets. He charges \$25.00 for the bracelet and then \$6.00 for each bead. Write an equation that represents the cost, given  $x$  number of beads that are chosen.

$$C = 6x + 25$$

- ③ The cost of operating Hannah's Biscotti Company is \$1600 per week plus \$.10 to make each biscotti cookie.
- Write a function,  $C(b)$ , to model the company's weekly costs for producing  $b$  biscotti. (See page 102)  
 $C(b) = 1600 + 0.10b$
  - What is the total weekly cost in dollars if the company produces 4,000 biscotti cookies?  
 Total weekly cost  
 $C(b) = 1600 + 0.10(4000) = \$2000$
  - Hannah's company makes a gross profit of \$0.60 for each biscotti cookie they sell. If they sold all 4000 biscotti they made, would they make money or lose money for the week (net profit)? How much?  
 Find gross profit before calculating  
 $Gross\ Profit = 0.60b$   
 $Gross\ Profit = 0.60(4000) = \$2400$   
 $Net\ Profit = Gross\ Profit - Cost$   
 $Net\ Profit = 2400 - 2000 = \$400$   
 The company would make \$400 profit for the week.

**Number Problems:** Use  $x$  for the number you know least about. Make the other parts of the "let" statement related to that one.

**Example** "One number is 21 less than twice the other number"  
 Their sum is 54. Find both numbers

- Steps:**
- 1) Let  $x$  = one number and let  $2x - 21$  = the other number.
  - 2) This is the equation:  $x + (2x - 21) = 54$
  - 3) Then solve as usual:  $3x - 21 = 54$   

$$\begin{array}{r} 3x - 21 = 54 \\ +21 \quad +21 \\ \hline 3x = 75 \\ x = 25 \end{array}$$
  - 4)  $x$  is:  $x = 25$
  - 5) Substitute the value of  $x$  to find the other number:  $2x - 21 \Rightarrow 2(25) - 21 = 29$
  - 6) **Answer:** 25 and 29

## Special Number Problems - sample "let statements"

**Consecutive Integer problems:** Let  $x$  = the first consecutive integer.  
 $x + 1$  = the 2<sup>nd</sup> consecutive integer and  $x + 2$  = the 3<sup>rd</sup>.

**Positive or Negative Consecutive Integers:** Let statement is still  $x$ ,  $x + 1$ ,  $x + 2$ , etc. Whether + or -, this "let" statement will work for all consecutive integer problems.

**Odd or Even Consecutive integers:** Let  $x$  = first odd integer,  $x + 2$  = next consecutive odd integer. Even integers have the SAME let statement. Again, positive or negative odd/even integers have this same "let" statements. Remember zero is considered an even number.

**Coin Problems:** There are two things to consider in these problems. You must determine whether the problem is giving the NUMBER of coins (how many coins there are) or the VALUE of the coins (what the coins are worth in money). USUALLY THE “LET” STATEMENT tells about the NUMBER OF COINS, NOT THEIR VALUE. The value is often used in making the equation. When setting up the equation, if the problem uses dollars then use the decimal values (0.05, 0.10, 0.25) for the coins. If the problem is given in cents, use 5 for nickels, 10 for dimes, 25 for quarters. You may not need values: sometimes the problem asks only about how many coins there are and does not include any information about value.

**Example** Joe has \$2.50. He has 7 more dimes than nickels.  
How many of each does he have?

**Steps:**

- 1) Set up the “Let” statement: Let  $x$  = number of nickels
- 2) Determine known values:  $\therefore x + 7$  = number of dimes
- 3) Make equation and insert values of monies:  $.05x + .10(x + 7) = 2.50$
- 4) Multiply out (distributive property):  $.05x + .10x + .70 = 2.50$
- 5) Isolate  $x$ :  $.15x + .70 = 2.50$   

$$\begin{array}{r} .15x + .70 = 2.50 \\ - .70 \quad - .70 \\ \hline .15x = 1.80 \\ .15 \quad .15 \\ \hline x = 12 \end{array}$$
- 6) Solve for  $x$ :  $x = 12$  nickels
- 7) Answer:  $x = 12$  nickels
- 8) Plug answer back into “Let” statement:  $x + 7 = 19$  dimes
- 9) Answer: He has 19 dimes and 12 nickels

**Note:**  $\therefore$  means “therefore”

**Ratio Problems:** Word problems sometimes involve ratios between numbers or items. Use the ratio information given in the problem to make the “let” statement. Then use the “let” statement to make an equation for the solution.

**Example** Find the measure of each angle of a triangle whose angles have a ratio of 3:6:9. (Use information you already know about triangles to form the equation: the sum of the angles in a triangle is  $180^\circ$ .)

**Steps:**

- 1) Set up the “Let” statement: Let  $3x$  = one angle,  $6x$  = the 2<sup>nd</sup> angle, and  $9x$  = the 3<sup>rd</sup> angle.
- 2) Make the equation:  $3x + 6x + 9x = 180$
- 3) Solve for  $x$ :  $18x = 180^\circ$  or  $x = 10$
- 4) Plug the answer in the “Let” statement to find the three angles:  $(10) = 30$ ;  $6(10) = 60$ ; and  $9(10) = 90$
- 5) Answer: The three angles measure  $30^\circ$ ,  $60^\circ$ , and  $90^\circ$ .

**Age and Mixture Problems:** Make a chart to show the information in the problem. This can be used as the “let” statement (or you can use it to make your written “let” statement). Make columns for the information given: Use the chart below to make an equation.

**Examples**

- ① Sue is 5 years older than Ann. In 6 years, Sue’s age will be 11 years less than twice Ann’s age then. How old is each person now?

Let Statement

NAME	AGE NOW	AGE IN 6 YEARS
Ann	$x$	$x + 6$
Sue	$x + 5$	$(x + 5) + 6$

**Steps:**

- 1) Analyze the phrases mathematically and make a chart:  
In 6 years (use “age in 6 years” column) Sue’s age  $[(x + 5) + 6]$  will be  $(=)$  11 years less than  $(-11)$  twice (2) Ann’s age then  $(x + 6)$ .
- 2) Make an equation:  $(x + 5) + 6 = 2(x + 6) - 11$
- 3) Use distributive property:  $x + 11 = 2x + 12 - 11$
- 4) Solve for  $x$ :  $x + 11 = 2x + 1$   

$$\begin{array}{r} -x \quad -1 \quad -x \quad -1 \\ \hline 10 = x \end{array}$$
- 5) Ann’s current age:  $10 = x$
- 6) Plug  $x$  back in to find Sue’s age:  $x + 5 = (10) + 5$ , so Sue’s age = 15
- 7) **Answer:** Ann is now 10 and Sue is now 15.
- 8) Check: In 6 years Ann will be 16, Sue will be 21 (21 is eleven less than twice 16).

- ② The 9<sup>th</sup> grade students in Public School #4 are selling tickets to the class play. Tickets purchased by students will cost \$3.00 each and tickets sold to the public will cost \$5.00 each. They sell 400 tickets but do not keep track of whether they are purchased by students or by the public. They have \$1500 after selling the tickets. How many of each type of ticket did they sell?

Type of Ticket	Tickets Sold	Price per Ticket	Money Earned
Student	$x$	\$3.00	$\$3.00x$
Public	$400 - x$	\$5.00	$\$5.00(400 - x)$
Total	400		\$1500

$$\begin{array}{r} 3x + 5(400 - x) = 1500 \\ 3x + 2000 - 5x = 1500 \\ -2000 \quad \quad -2000 \\ \hline -2x = -500 \\ x = 250 \\ 400 - x = 400 - 250 = 150 \end{array}$$

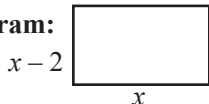
**Answer:** 250 tickets were sold to students and 150 were sold to the public.

**Geometry Word Problems:** Draw a diagram to demonstrate the problem.

Label it with the information for the “let” statement. Substitute what is given in the formula.  
(Example on top of next page.)

**Example** A garden has a perimeter of 48 feet. If one side of the garden is 2 feet shorter than the other side, what are the dimensions of the garden?

**Diagram:**



or use a "let" statement

Let  $x$  = the length

Let  $x - 2$  = the width

**Formula:**  $P = 2(l + w)$

$$48 = 2(x + x - 2)$$

$$48 = 2(2x - 2)$$

$$48 = 4x - 4$$

$$+4 \quad +4$$

$$52 = 4x$$

$$\frac{52}{4} = \frac{4x}{4}$$

$$13 = x$$

$$x - 2 \Rightarrow 13 - 2 = 11$$

**Conclusion:** The garden is 13 ft by 11 ft.

### PERCENTS – INCREASE, DECREASE, AND DISCOUNT

**Discount:** Stores often give a % off as a discount. Find the sale price by using this: *Original selling price – Discount amount = Discount sale price*

**Note:** The dollar sign is usually omitted in the formula but must be put back in the answer when the answer is in dollars.

#### **Examples**

- ① Jerome bought a pair of sneakers at a 25% discount sale. He paid \$60.00. What was the original price of the sneakers?

Original Price =  $x$

Discount amount =  $0.25x$  (Change the % to a decimal.)

Sale Price = \$60

Substitute in the formula:

$$x - 0.25x = 60$$

$$0.75x = 60$$

$$x = 80 \text{ The original price was } \$80.$$

- ② Find the cost of a \$200 bike that is on sale at 15% off.

Original Price = \$200, Discount Amount =  $(0.15)(\$200)$ ,

Discount sale price =  $x$

$$200 - (0.15)(200) = x \text{ Substitute}$$

$$200 - 30 = x \text{ Solve for } x$$

$$x = 170 \text{ The discount sale price of the bike is } \$170.$$

- ③ What is the discount on a skateboard that was originally sold for \$150 and is now on sale for \$120?

Original Price = \$150, Discount Amount =  $(x)(\$150)$ ,

Discount sale price = \$120

$$150 - 150x = 120 \text{ Substitute}$$

$$-150x = 120 - 150$$

$$x = \frac{-30}{-150} \text{ Solve}$$

$$x = 0.2 \text{ (Change to a \% by multiplying by 100\%)}$$

The discount is 20% off.

**Decrease:** Another way to say this is that the sale price is a *decrease of 20%* from the original price. This is called a percent of decrease.

**(Examples continued from previous page.)**

- ④ My pool was  $72^{\circ}\text{F}$  on Monday. On Friday it was  $80^{\circ}\text{F}$ . What is the **percent of increase**, to the nearest tenth, of the temperature in the pool? This is not a money problem, but it can be handled in a similar way. Since the temperature is increasing, use  $+$  the change for the 2<sup>nd</sup> term.

Original + change = final result

Original temperature (72) + % change ( $x$ ) = Final temperature (80)

$$72 + (x)(72) = 80$$

$$72x = 8$$

$$x = 0.1111\ldots \quad \text{Change to \%} = 11.111\ldots\%$$

The pool temperature increased by about 11.1%

**Similarity:** 2 polygons are similar if they have corresponding sides that are proportional and corresponding angles that are congruent. The order in which the letters identifying the vertices in one polygon are written to match the order of the corresponding vertices of the 2<sup>nd</sup> polygon. To find the lengths of sides of similar polygons, we use proportions.

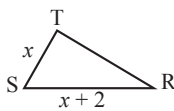
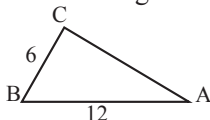
**Solving Geometry Problems with Algebraic Fractions and Proportions**

Problems using proportions:

1. Draw diagrams of both similar figures and label the known sides.
2. Make two equal ratios using information from one polygon for the numerators of both fractions and corresponding information from the other polygon as the denominators of both fractions.
3. Multiply the numerator of one fraction by the denominator of the other. Then multiply the denominator of the first fraction by the numerator of the second fraction. This is commonly called “cross multiply”. Solve.

**Example**  $\triangle ABC$  is similar to  $\triangle RST$ . If  $AB = 12$  and  $BC = 6$ , find the lengths of  $RS$  and  $ST$  if  $RS$  is 2 units more than  $ST$ .

**Steps 1)**



2) Match up similar sides:  $\overline{AB} \sim \overline{RS}$ ,  $\overline{BC} \sim \overline{ST}$

3) Set up the proportion:  $\frac{6}{x} = \frac{12}{x+2}$

$$6(x+2) = 12x$$

4) Solve for  $x$ :

$$6x + 12 = 12x$$

$$\begin{array}{r} -6x \qquad \qquad -6x \\ \hline 12 = 6x \end{array}$$

$$x = 2$$

5) Conclusion:

$$ST = 2, \text{ and } RS = 4$$

**Scale Drawings:** A scale drawing is a reduction or an enlargement of a real object. Architectural drawings, models, and maps are some examples of scale drawings.

**Scale:** The RATIO in the drawing. Use the ratio that is the scale of the drawing in a proportion to find information about the drawing or the real object.

General Proportion:  $\text{Scale (a fraction)} = \frac{\text{Drawing}}{\text{actual or real object}}$

**Example** The scale of a map is 1 cm to 5 km. The distance on the map from Santa Fe to Johnstown is 4.5 cm. What is the actual distance between these two cities?

The SCALE is 1 cm : 5 km., the distance in the map or “drawing” is 4.5 cm, and we don’t know the real object distance.

**Steps** 1) Use a variable for the real object distance and make a proportion:  $\frac{1\text{cm}}{5\text{km}} = \frac{4.5\text{cm}}{x}$

2) Cross multiply:  $1\text{cm} \cdot x = (4.5\text{cm})(5\text{km})$

3) Divide by 1 cm to simplify:  
(In this step, the “cm” cancels.)  $\frac{1\text{cm} \cdot x}{1\text{cm}} = \frac{(4.5\cancel{\text{cm}})(5\text{km})}{1\cancel{\text{cm}}}$

4) Answer:  $x = 22.5\text{ km}$

**Note:** Include the units of measure. Some will cancel while the work is being performed, giving the correct units for the final answer. The actual distance from Santa Fe to Johnstown is 22.5 km.

**Exponential Equations:** An exponential equation has a variable in the exponent. For now, these problems can be solved by making a table or a graph.

**Example** Nathan owns 4 rabbits. He expects the number of rabbits to double every year. After how many years will he have 64 rabbits?

Let  $x$  = number of years

Let  $y$  = number of rabbits

Write an equation to model this situation.

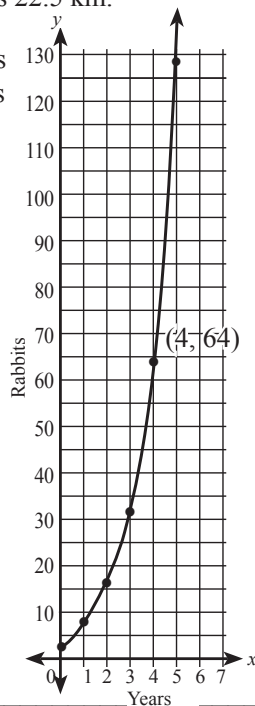
**Answer:**  $y = 4(2)^x$

Use a table or a graph to solve the equation. [The use of a grid is optional.]

**Answer:** 4 years and appropriate work is shown.

(See also Unit 4)

$x$	$y$
0	4
1	8
2	16
3	32
4	64
5	128





# CHARACTERISTICS OF FUNCTION GRAPHS

**Type of Function:** There are many different types of functions. The types of functions discussed here include constant, linear or identity, quadratic, exponential, square root, cube root, absolute value, and piecewise-defined including step functions.

**Types of Graphs:** There are certain shapes of graphs that are associated with various functions. The general shape of these graphs will be similar although the size and exact shape may vary based on the specific numbers used in the function. The basic functions are called *parent functions*. It is easy and fun to use a graphing calculator to change the size, shape, and location of the graphs of these functions by changing the coefficients of the variables and making other changes in the basic function.

**Domain:** Remember the domain is the set of values of  $x$  that are available for input to the function.

**Range:** The range is the set of possible output values,  $y$  or  $f(x)$ .

**Increasing:** For any  $x_1$  and  $x_2$  where  $x_1 < x_2$ , if  $f(x_1) < f(x_2)$  then the graph is increasing in the interval between those points.

**Decreasing:** For any  $x_1$  and  $x_2$  where  $x_1 < x_2$ , if  $f(x_1) > f(x_2)$  then the graph is decreasing in the interval between those points.

**Constant:** For any  $x_1$  and  $x_2$  where  $x_1 < x_2$ , if  $f(x_1) = f(x_2)$  then the graph is constant in the interval between those points. This would be a horizontal line.

**Function 1**

This function is:

**Decreasing**

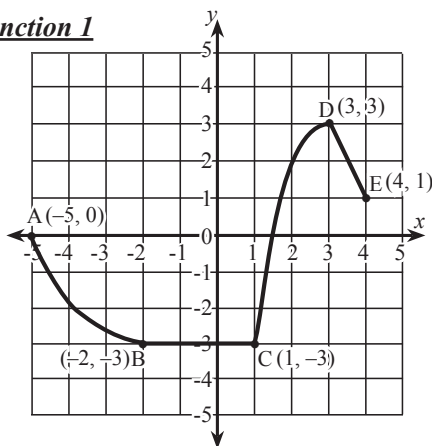
between  $x = -5$  and  $x = -2$   
and between  $x = 3$  and  $x = 4$ .

**Increasing**

between  $x = -2$  and  $x = 3$ .

**Constant**

between  $x = -2$  and  $x = 1$ .



**Vertex:** In a parabola, the highest or lowest point. Also called the turning point.

**Maxima or Minima:** The value of  $f(x)$  at the vertex. A maximum point is the point at which the graph changes increasing to decreasing. The minima, or minimum point, is where the graph changes from decreasing to increasing.

**Relative Maxima or Minima:** If a graph has more than one turning point, relative maxima or minima are the values of  $f(x)$  at the points where the graph changes from decreasing to increasing and vice versa.

**Symmetry:** Some functions, but not all, will be symmetric to a line.

**Intercepts:** The point(s) at which the graph intersects the  $x$ -axis or  $y$ -axis.

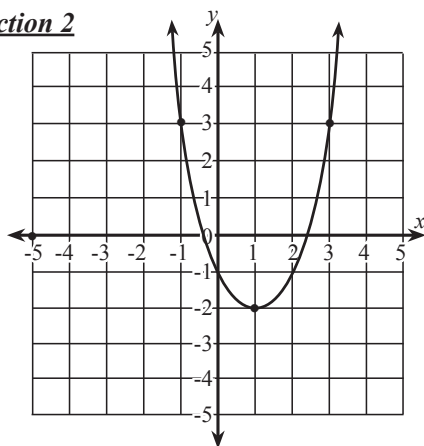
**Function 2**

**Vertex:**  $(1, -2)$

**Minima:**  $y = -2$

**Symmetric** to the line  $x = 1$

**Intercepts:**  $x = -\frac{1}{2}, x = 2\frac{1}{2}$   
 $y = -1$

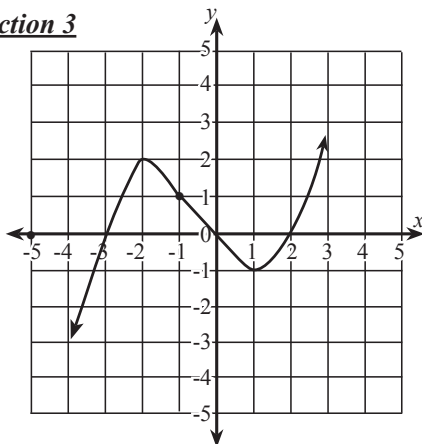


**Function 3**

**Relative maxima:**  $y = 2$

**Relative minima:**  $y = -1$

**Intercepts:**  $x = -3, x = 0,$   
 $x = 2, y = 0$

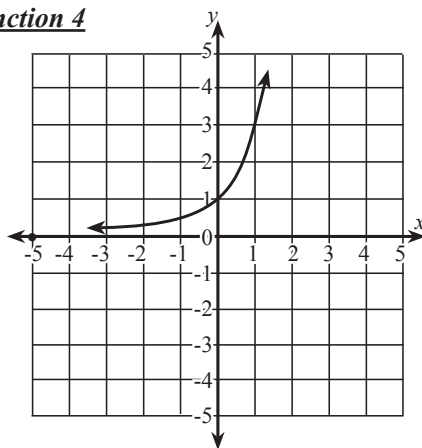


**Asymptote:** A line that is approached by a graph as it approaches infinity.

**Function 4**

**Asymptote:**  $y = 0$

**Intercepts:**  $y = 1$

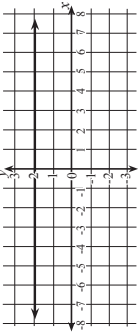
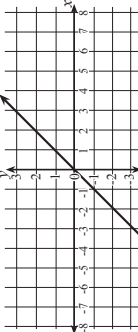
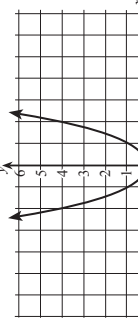
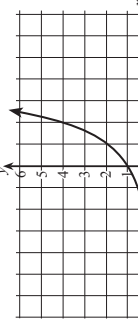


**Evaluate:** If asked to evaluate  $f(x)$  from a graph, locate the given  $x$  value on the horizontal axis and read the corresponding  $y$  value. In function 4, evaluate  $f(1)$ . At  $x = 1$ ,  $y = 2$ . Therefore,  $f(1) = 2$ .

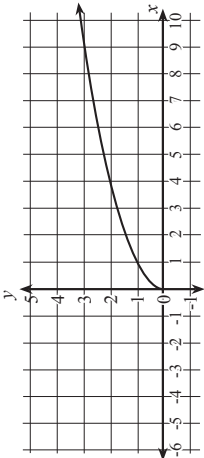
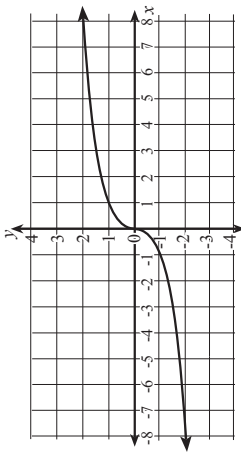
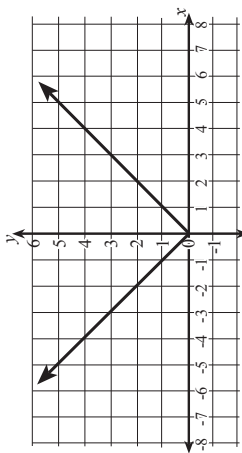
**Parent Function:** The basic form of a particular type of function. Examples given are the parent function where possible.

**Note:** Points read from a graph are approximated. Determine the numbers as closely as possible.

# Linear, Quadratic and Exponential Parent Functions

Type of $f(x)$	Parent Function	Sketch	Intercepts	Max/Min	Notes
<b>Constant</b>	$f(x) = b$ <i>Note:</i> $f(2)$ is used for the sketch.		y-intercept (0, 2). Parallel to the x-axis, no x-intercept.		
<b>Identity</b> (This is the basic form of a linear equation in the form $y = mx + b$ )	$f(x) = x$		(0, 0)		Linear and Identity function relationship: The Identity Function is the familiar slope intercept linear equation where $m = 1$ and $b = 0$ .
<b>Square or Quadratic</b>	$f(x) = x^2$		(0, 0)	Min: (0, 0)	
<b>Exponential</b>	$f(x) = b^x$ ; $b > 0, b \neq 1$ <i>Note:</i> $b = 2$ is used for the sketch. If $0 < b < 1$ , the graph is reflected over the y-axis.		y-intercept at (0, 1). No x-intercept.	The x-axis is an <i>asymptote</i> for the graph of this function.	The y-intercept is (0, 1). There is no x-intercept. If $b > 1$ , the exponential graph is increasing. If $0 < b < 1$ the graph is decreasing.

# **Square Root, Cube Root and Absolute Value Parent Functions**

Type of $f(x)$	Function Rule	Sketch	Intercepts	Max/Min	Notes
<b>Square Root</b>	$f(x) = \sqrt{x}, x \geq 0$		(0, 0)	Minimum (0, 0)	Domain: $x \geq 0$ . Negative numbers do not have a real number square root. Range: $y \geq 0$ . The $\sqrt{\phantom{x}}$ symbol indicates the principal root only.
<b>Cube Root</b>	$f(x) = \sqrt[3]{x}$		(0, 0)		The domain and range of this function are both all the real numbers.
<b>Absolute Value</b>	$f(x) =  x $		(0, 0)	Minimum (0, 0)	Domain: Real numbers. Range: $y \geq 0$ This function is also considered a piecewise-defined function. $f(x) = \begin{cases} -x & \text{if } x < 0 \\ x & \text{if } x \geq 0 \end{cases}$

**Piecewise-Defined Functions:** Piecewise functions have at least two different rules that are used for different parts of their domain. Absolute value functions, step functions, and other functions that are piecewise are shown here. There may be different rules applied to separate parts of the domain.

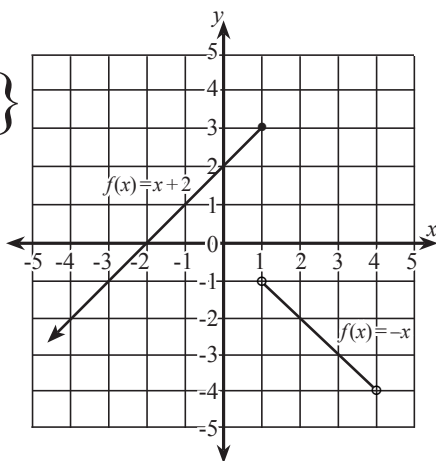
The rules for the function are written using large brackets with several rules inside. The word “if” is sometimes but not always used. To evaluate a piecewise function for a value of  $x$ , determine which part of the domain  $x$  is in and use the rule that is applied to that part.

**Piecewise Function:** This means that from the domain of  $-\infty$  to and including 1, the function is  $f(x) = x + 2$ . The next part of the function is  $f(x) = -x$  in the domain 1 to 5 but not including 1 or 5.

**Examples**

$$\textcircled{1} \quad f(x) = \begin{cases} x + 2 & \text{if } x \leq 1 \\ -x & \text{if } 1 < x < 5 \end{cases}$$

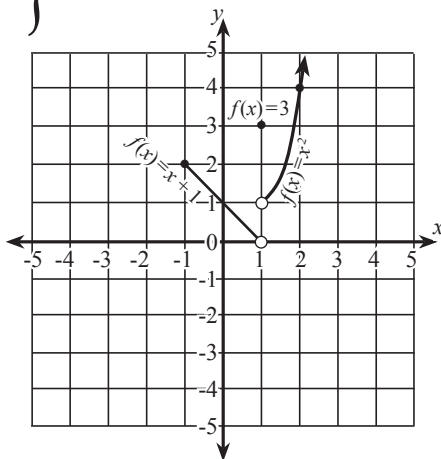
This means that from the domain of  $-\infty$  to and including 1, the function is  $f(x) = x + 2$ . The next part of the function is  $f(x) = -x$  in the domain 1 to 4 but not including 1 or 4.



$$\textcircled{2} \quad f(x) = \begin{cases} -x + 1 & -1 \leq x < 1 \\ 3 & x = 1 \\ x^2 & x > 1 \end{cases}$$

**Domain:**  $x \geq -1$

**Range:**  $y > 0$



**Step Function:** Step functions are used for certain situations in which the graph has a constant value over sections of its domain. An example would be if every non-integer value of  $x$  down to the next lower integer to obtain  $f(x)$ . This is called a “greatest integer” function – think of it as the largest or greatest integer less than the given number. The symbol for the greatest integer is  $\llbracket x \rrbracket$ . The graph of a step function looks like a staircase viewed from the side! Sometimes the steps are connected with vertical lines, and sometimes they remain separate from each other.

**Step Function:**  $f(x) = \llbracket x \rrbracket$

$$f(x) = \left\{ \begin{array}{l} \dots -2 \text{ if } -2 \leq x < -1 \\ -1 \text{ if } -1 \leq x < 0 \\ 0 \text{ if } 0 \leq x < 1 \\ 1 \text{ if } 1 \leq x < 2 \dots \end{array} \right\}$$

The “...” means to continue in the same pattern in the negative direction and in the positive direction.

**Domain:** Real numbers

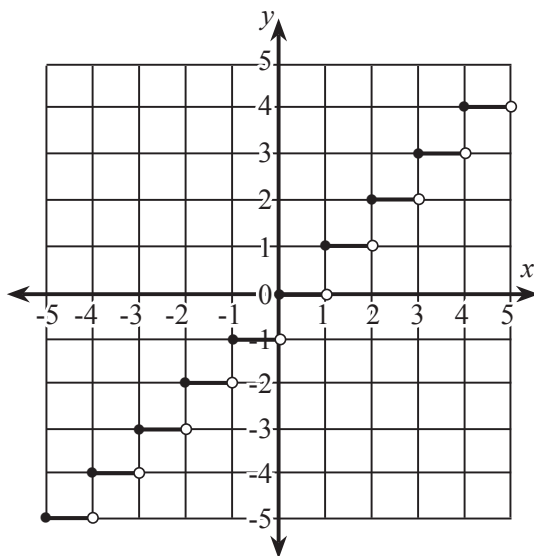
**Range:** Integers

**Evaluate:**

$$f\left(-\frac{1}{3}\right) = -1$$

$$f(2) = 2$$

$$f(2.75) = 2$$





**Review of Parent Functions**Identity (Linear):  $f(x) = x$ Radical:  $f(x) = \sqrt{x}$ Quadratic:  $f(x) = x^2$ Cubic:  $f(x) = x^3$ Exponential:  $f(x) = b^x$ Absolute Value:  $f(x) = |x|$ **Rules for Transformations of Functions** $f(x)$  represents the parent function

$f(x) = x^2$  and the number 3 is used to demonstrate the rules on this page.  
For more parent functions and their transformations,  
see the next 3 pages.

 $f(x) + a$  : moves the graph up  $a$  units.

Ex:  $f(x) + 3 = x^2 + 3$

 $f(x) - a$  : moves the graph down  $a$  units.

Ex:  $f(x) - 3 = x^2 - 3$

 $f(x + a)$  : moves it to the left  $a$  units.

Ex:  $f(x + 3) = (x + 3)^2$

 $f(x - a)$  : moves it to the right  $a$  units.

Ex:  $f(x - 3) = (x - 3)^2$

 $-f(x)$  : reflects the graph over the  $x$ -axis.

Ex:  $-f(x) = -x^2$

 $f(-x)$  : reflects the graph over the  $y$ -axis.

Ex:  $f(x) = (-x)^2$

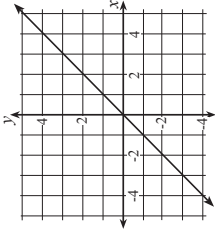
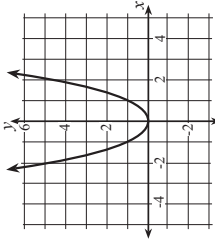
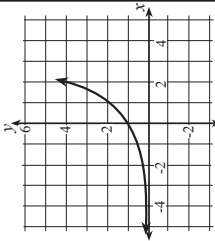
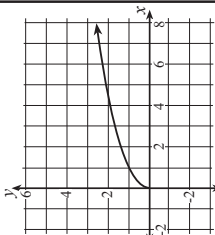
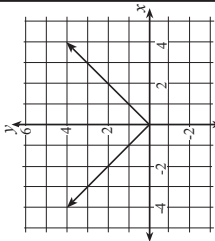
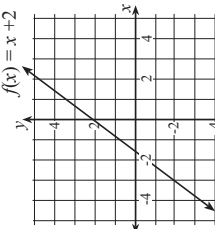
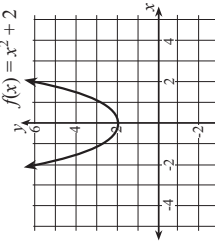
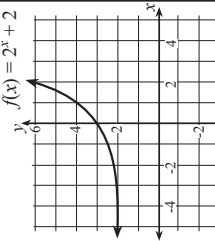
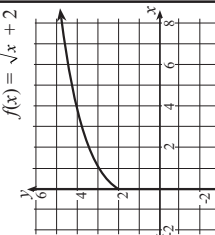
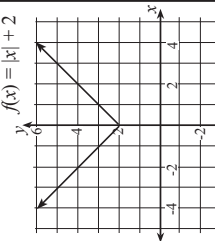
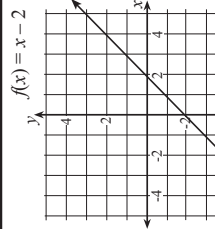
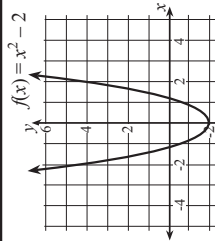
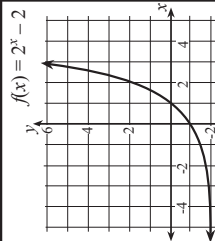
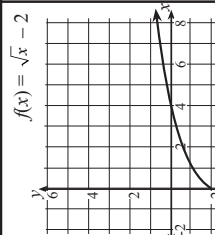
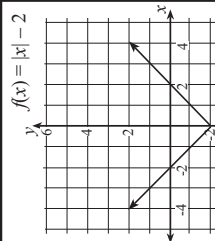
$a \cdot f(x)$  : stretches the graph vertically (or  
compresses it horizontally) when  $a > 1$ .

Ex:  $f(x) = 3x^2$

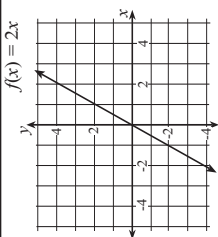
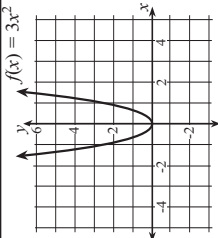
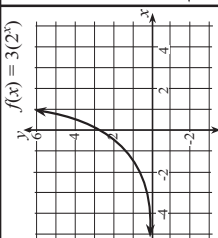
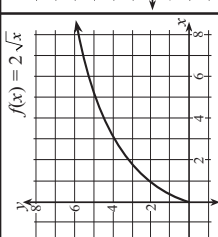
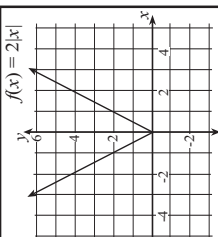
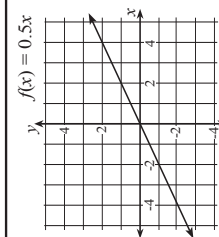
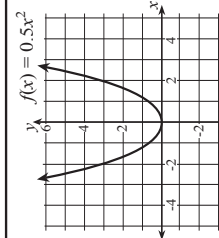
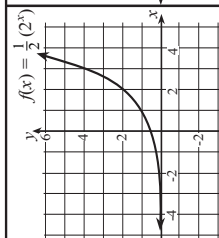
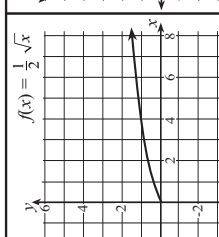
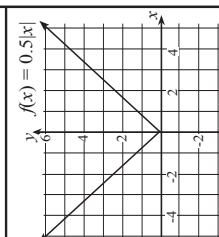
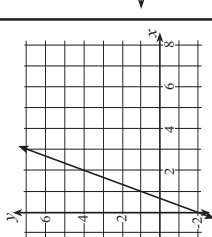
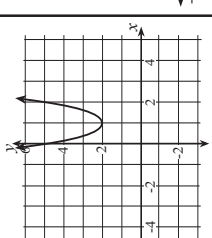
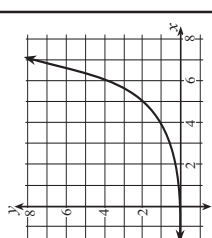
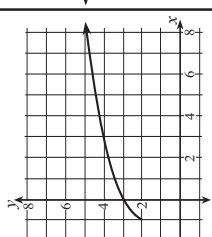
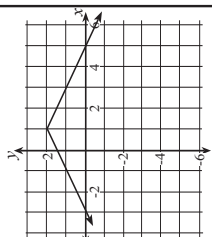
$a \cdot f(x)$  : compresses the graph vertically (or  
stretches it horizontally) when  $0 < a < 1$ .

Ex:  $f(x) = 0.5x^2$

## EFFECTS OF TRANSFORMATIONS ON PARENT FUNCTION GRAPHS

Parent Function	Identity (Linear) $f(x) = x$ $f(x) = mx + b$	Quadratic $f(x) = x^2$	Exponential $f(x) = b^x, b > 0, b \neq 1$ (this example, $b = 2$ )	Square Root $f(x) = \sqrt{x}, x \geq 0$	Absolute Value $f(x) =  x $
Parent Graph					
$f(x) + a$ Graph moves up $a$ units.					
$f(x) - a$ Graph moves down $a$ units.					

$f(x+a)$ <b>Graph moves LEFT <math>a</math> units.</b>	$f(x) = (x+2)^2$ 	$f(x) = 2^{(x+2)}$ 	$f(x) = \sqrt{x+2}$ 	$f(x) =  x+2 $ 
$f(x-a)$ <b>Graph moves RIGHT <math>a</math> units.</b>	$f(x) = (x-2)^2$ 	$f(x) = 2^{(x-2)}$ 	$f(x) = \sqrt{x-2}$ 	$f(x) =  x-2 $ 
$-f(x)$ <b>Graph is reflected over the <math>x</math>-axis.</b>	$f(x) = -x^2$ 	$f(x) = -(2^x)$ 	$f(x) = -\sqrt{x}$ 	$f(x) = - x $ 
$f(-x)$ <b>Graph is reflected over the <math>y</math>-axis.</b>	$f(x) = (-x)^2$ 	$f(x) = 2^{-x}$ 	$f(x) = \sqrt{-x}$ 	$f(x) =  -x $ 

$a \cdot f(x)$ $a > 1$ <b>Graph stretches up vertically or compresses horizontally.*</b>	$f(x) = 2x$ 	$f(x) = 3x^2$ 	$f(x) = 3(2^x)$ 	$f(x) = 2\sqrt{x}$ 	$f(x) = 2 x $ 
$a \cdot f(x)$ $0 < a < 1$ <b>Graph compresses vertically or stretches horizontally.*</b>	$f(x) = 0.5x$ 	$f(x) = 0.5x^2$ 	$f(x) = \frac{1}{2}(2^x)$ 	$f(x) = \frac{1}{2}\sqrt{x}$ 	$f(x) = 0.5 x $ 
<b>*Vertical stretch is also called horizontal compression. Vertical compression is also called horizontal stretch.</b>					
<b>New <math>f(x)</math></b>	$f(x) = 3x - 2$ 	$f(x) = 3(x - 1)^2 + 2$ 	$f(x) = 0.5(2^{x-3})$ 	$f(x) = \sqrt{x + 1} + 2$ 	$f(x) = -0.5 x - 1  + 2$ 
<b>Effects on parent graph</b>	<b>Graph moves down 2, stretches vertically – slope becomes 3.</b>	<b>Graph moves right 1, up 2, and stretches vertically making it more narrow.</b>	<b>Graph compresses vertically 0.5 and moves 3 units to the right.</b>	<b>Graph moves one unit to the left and 2 units up.</b>	<b>Graph reflects over x-axis, horizontally stretches (vertically compresses) by 0.5, moves right 1, and moves up 2.</b>

**DISPLAYING BIVARIATE DATA (2 VARIABLES)**

The display of bivariate data depends on the type of data, quantitative or categorical, that is contained in the data set. Quantitative is discussed below. Categorical bivariate data can be displayed using a two-way frequency table. (See page 169)

**Quantitative Data** with two variables can be displayed using a **scatter plot**.

In this numerical data, the values for the two variables are paired, much like  $(x, y)$  in coordinate graphing. Each pair of values becomes a point that is recorded on a grid using the horizontal axis for the independent value,  $x$ , and the vertical axis for the dependent variable,  $y$ . The graph displays the relationship between the independent and dependent variables. The plotted points often suggest a pattern (a line or a curve) which can be described using a function. Although producing a scatter plot and doing associated work with it can be done without a graphing calculator, it is recommended that one be used.

**Line (or Curve) of Best Fit:** It is a sketch through the points on the scatter plot such that the data points are distributed as equally as possible on both sides of the line or curve. A function (or equation) can be written to describe the line of best fit. This equation is also called a regression equation. In the example below, “Feeding the Birds”, it seems that a line could be sketched through the data so the data points are equally distributed on each side of the line. The line can be defined using a linear function. (Sometimes a curve is needed. See page 167.)

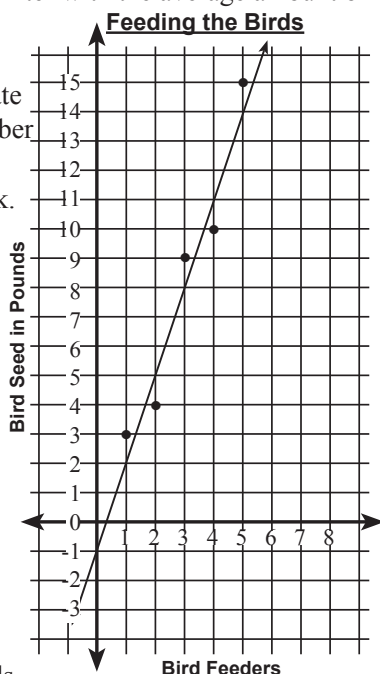
**Regression Equation:** A regression equation is a function that describes the data. It is also called the line (or curve) of best fit. Examine the scatter plot to see if the data appear to cluster around a line or curve. A regression equation can be written to describe the line or curve either manually or using a graphing calculator. The regression equation can also be used to predict the location of a data point not included in the observation.

**Example** Several neighbors were comparing the number of bird feeders that they have in their yards in the winter with the average amount of bird seed they used in a week.

**Steps**

- 1) Create a scatter plot to demonstrate the relationship between the number of bird feeders and the pounds of bird seed used in an average week.
- 2) Sketch a line or curve of best fit. Determine a function to define it.
- 3) Describe the relationship of the independent and dependent variables.

Bird Feeders	Bird Seed (pounds)
1	3
2	4
3	9
4	10
5	15



**Solution:**

- a) The scatter plot for Feeding the Birds is sketched. Appropriate labels are included.
- b) The data appear to cluster along a line. Sketch a line as accurately as possible between the data - some points maybe on it, some will not. After the line is sketched, it is necessary to define a function to describe this line.
- c) When examining the data, as the independent variable ( $x$ ) increases the dependent variable ( $y$ ) also increases. This shows an upward or positive trend of the data.

**Note:** The line of best fit can be sketched by hand although the graphing calculators will make this process easier. Without the calculator the work needs to be done manually. Write an *approximate* equation of the line of best fit in the form  $y = mx + b$ .

## FINDING THE APPROXIMATE REGRESSION EQUATION

### Method Without A Calculator:

#### Steps

- 1) Choose two plotted points on the sketch that are close to or on the line of best fit. Use the slope formula,  $m = \frac{y_2 - y_1}{x_2 - x_1}$ , to find the slope of the line.
- 2) Then use the point-slope formula,  $y - y_1 = m(x - x_1)$ , to complete the equation. Write the equation  $y = mx + b$  form.

**Example** The points closest to the line of best fit appear to be (1, 3) and (3, 9).

- 1)  $m = \frac{9-3}{3-1} = \frac{6}{2} = 3$  slope
- 2)  $y - 3 = 3(x - 1)$   
 $y - 3 = 3x - 3$   
 $y = 3x$  approximate equation

**Method With A Calculator:** The graphing calculator will provide a more accurate equation describing the line of best fit. When calculated using the appropriate features on the graphing calculator, the equation of the line of best fit is  $y = 3x - 0.8$ . This is a more accurate estimation than the one done manually. The calculator equation will be used for the remaining work on this problem.

**Calculator Screens:** Shown here are the screens involved in doing this work using the TI-84 Plus Calculator.

#### Steps

1)

L3	L4	L5	5
1	3		
2	4		
3	9		
4	10		
5	15		
-----			
15(1)=			

2)

```

LinReg
y=ax+b
a=3
b=-.8
r²=.9493670886
r=.9743547037
  
```

3)

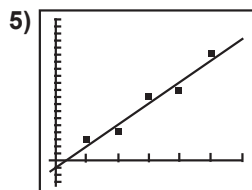
```

LinReg(ax+b)
Xlist:L3
Ylist:L3
FregList:
Store RegEQ:Y1
Calculate
  
```

4)

```

Plot1 Plot2 Plot3
Y1=X+.8
Y2=
Y3=
Y4=
Y5=
Y6=
Y7=
  
```





**Prediction:** The regression function can be used to *predict other points* within or beyond the observed data.

- **Interpolation:** Prediction within the observed data. If it were possible to have  $2\frac{1}{2}$  bird feeders, substitute 2.5 for  $x$  in the regression function.  $f(x) = 3x - 0.8$ . The amount of seed predicted would be about 6.7 lbs.
- **Extrapolation:** Prediction of a point beyond the observed data. To see how many pounds of bird seed should be needed for 50 bird feeders, substitute 50 for  $x$  in the equation  $f(x) = 3x - 0.8$ . The bird seed needed would be approximately 149.2 pounds.

**Slope and y-intercept** of the linear regression line with regard to the data must be considered. The regression equation calculated using the calculator is  $y = 3x - 0.8$ . **The slope is 3 and the y-intercept is  $-0.8$ .** Considering the type of data we collected, we know that a y-intercept of  $-0.8$  is not possible. We know that we cannot have a negative value of a quantity of birdseed. (Remember this is an approximate equation.) The slope of 3 indicates that as the number of bird feeders increases by one, the pounds of bird seed needed increases by 3. Since the person who only has one feeder uses 3 pounds of seeds, it is reasonable that for each additional feeder 3 more pounds of seed are needed.

**Correlation:** It is a number that indicates how closely the data are represented by the line or curve of best fit. The number is called the correlation coefficient and is represented using  $r$ . The correlation coefficient,  $r$ , is part of the work done within the calculator to find the correct regression equation for the function. It is always between  $-1$  and  $+1$ . The closer  $r$  is to  $-1$  or to  $+1$ , the stronger the correlation. A weak correlation is shown by an  $r$  value close to zero, either positive or negative.

**Example** In the bird feeder example. What is the correlation coefficient between the data and the line of best fit as determined by the calculator?

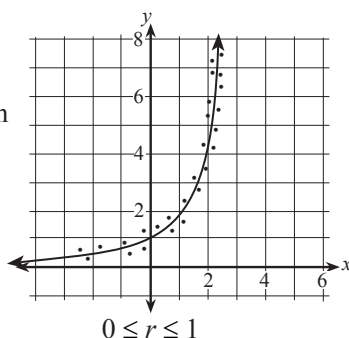
**Solution:** Use the statistics functions in your calculator to find the value of  $r$ .

The correlation coefficient  $r = 0.974354037$ . This number is quite close to  $+1$  and shows a strong correlation between the data and the regression line. It shows a good fit for the line of best fit!

**Positive or Negative Value of  $r$ :** The positive or negative value of  $r$  refers to the general trend of the data.

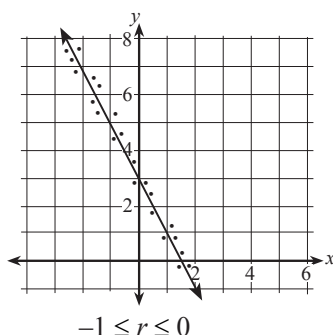
- **Positive (+) correlation** means the data have an upward tendency, left to right, when graphed. A graph with a positive correlation will have a regression equation with a positive slope or a positive rate of change (slope between any two points on the graph). The bird feeder example data has a positive trend – as the independent variable increases, the dependent variable also increases.

### Positive Correlation



- **Negative (–) correlation** means the data have a downward tendency when read left to right on the graphed. A graph with a negative correlation will have a regression equation with a negative slope or a negative rate of change (slope between any two points on the graph). As the independent variable increases, the dependent variable decreases.

### Negative Correlation



**Residual:** It is the difference between the observed  $y$  value ( $y_o$ ) and the predicted  $y$  value ( $y_p$ ) at each observed  $x$  value of ( $x_o$ ). The residuals are another way to examine the correlation of the line of best fit. Ideally the residuals should be as small as possible which would indicate that the regression line was a good fit for the data. The strength of the relationship between the data and the regression function is determined by examining the location of each plotted point compared with the line or curve of best fit.

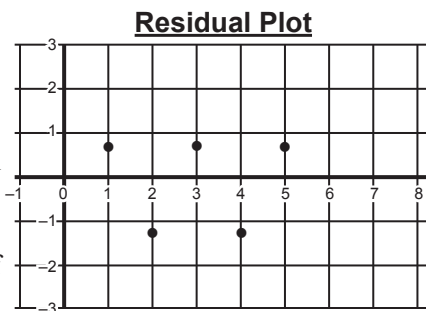
**Residual Plot:** It is a scatter plot of the points representing the differences between the observed  $y_o$ , and the predicted  $y_p$ , at each observed value of  $x_o$ . The plotted ordered pairs are ( $x_o, y_r$ ). The *residual* ( $y_r$ ) is the difference between an observed “ $y$ ” value ( $y_o$ ) and the “ $y$ ” value ( $y_p$ ) predicted by the equation for the line of best fit, also called the regression line. A good fit would be indicated if most of the points of the residual plot are near the line  $y = 0$ . The residuals can be used to discuss the correlation of the line of best fit.

**Example** Determine whether the line of best fit shown in the bird feeder example on page 164 is a good fit for the data. This is called the correlation.

**Plan:** Use the regression function found using the calculator for best accuracy. Substitute the observed values of  $x$ , ( $x_o$ ), in the function to find the corresponding predicted value of  $y_p$ . Find the difference between the predicted value of  $y_p$ , and the observed  $y_o$  value. We will call the difference,  $y_r$ . The calculation of  $y_r$  is:  $y_o - y_p = y_r$ .

Bird Feeders ( $x_o$ )	Bird Seed (lbs) ( $y_o$ )	$y_p = 3x_o - 0.8$	$y_o - y_p = y_r$	Residual Plot ( $x_o, y_r$ )
1	3	2.2	0.8	(1, 0.8)
2	4	5.2	-1.2	(2, -1.2)
3	9	8.2	0.8	(3, 0.8)
4	10	11.2	-1.2	(4, -1.2)
5	15	14.2	0.8	(5, 0.8)

**Analysis:** After plotting a graph of the residuals, we can see that most of the  $y$  points are close to  $y = 0$ . The observed points have small differences or residuals with respect to the line of best fit. This indicates that the line of best fit is a good fit. The data and the line of best fit have a strong correlation.



**Causation:** Even a strong positive or negative correlation does not necessarily imply cause and effect. In the bird feeder example, the number of pounds of bird seed used does appear to be caused by how many bird feeders a person has. In other cases a strong association could be caused by other variables. Concluding that “ $x$  causes  $y$ ” cannot be proved simply with the correlation coefficients and residuals.

**Conclusion:** There are many other quantitative statistical calculations that can be performed using data and their associated graphing calculator functions which will be studied in later math courses. Figure 1, on the previous page, is used to demonstrate a positive correlation. It represents an exponential regression which has a curved “line” of best fit. Other types of regression include logarithmic and power regressions, which are also both curved.

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