

# ALGEBRA 2

## Workbook

### Common Core Standards Edition

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**Common Core High School Math Reference Sheet  
(Algebra I, Geometry, Algebra II)**

**CONVERSIONS**

1 inch = 2.54 centimeters	1 kilometer = 0.62 mile	1 cup = 8 fluid ounces
1 meter = 39.37 inches	1 pound = 16 ounces	1 pint = 2 cups
1 mile = 5280 feet	1 pound = 0.454 kilograms	1 quart = 2 pints
1 mile = 1760 yards	1 kilogram = 2.2 pounds	1 gallon = 4 quarts
1 mile = 1.609 kilometers	1 ton = 2000 pounds	1 gallon = 3.785 liters
		1 liter = 0.264 gallon
		1 liter = 1000 cubic centimeters

**FORMULAS**

Triangle	$A = \frac{1}{2}bh$	Pythagorean Theorem	$a^2 + b^2 = c^2$
Parallelogram	$A = bh$	Quadratic Formula	$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$
Circle	$A = \pi r^2$	Arithmetic Sequence	$a_n = a_1 + (n-1)d$
Circle	$C = \pi d$ or $C = 2\pi r$	Geometric Sequence	$a_n = a_1 r^{n-1}$
General Prisms	$V = Bh$	Geometric Series	$S_n = \frac{a_1 - a_1 r^n}{1 - r}$ where $r \neq 1$
Cylinder	$V = \pi r^2 h$	Radians	1 radian = $\frac{180}{\pi}$ degrees
Sphere	$V = \frac{4}{3}\pi r^3$	Degrees	1 degree = $\frac{\pi}{180}$ radians
Cone	$V = \frac{1}{3}\pi r^2 h$	Exponential Growth/Decay	$A = A_0 e^{k(t-t_0)} + B_0$
Pyramid	$V = \frac{1}{3}Bh$		



August 2022

## Part I

Answer all 24 questions in this part. Each correct answer will receive 2 credits. No partial credit will be allowed. Utilize the information provided for each question to determine your answer. Note that diagrams are not necessarily drawn to scale. For each statement or question, choose the word or expression that, of those given, best completes the statement or answers the question. Record your answers in the space provided. [48]

1. The Hot and Tasty Coffee chain conducts a survey of its customers at its location at the Staten Island ferry terminal. After the survey is completed, the statistical consultant states that 70% of customers who took the survey said the most important factor in choosing where to get their coffee is how fast they are served. Based on this result, Hot and Tasty Coffee can infer that

(1) most of its customers in New York State care most about being served quickly

(2) coffee drinkers care less about taste and more about being served quickly

(3) most of its customers at the Staten Island ferry terminal care most about being served quickly

(4) most of its customers at transportation terminals and stations care most about being served quickly

1 \_\_\_\_\_

2. Given that  $i$  is the imaginary unit, the expression  $(x - 2i)^2$  is equivalent to

(1)  $x^2 + 4$       (2)  $x^2 - 4$       (3)  $x^2 - 2xi - 4$       (4)  $x^2 - 4xi - 4$

2 \_\_\_\_\_

3. The equation below can be used to model the height of a tide in feet,  $H(t)$ , on a beach at  $t$  hours.

$$H(t) = 4.8\sin\left(\frac{\pi}{6}(t + 3)\right) + 5.1$$

Using this function, the amplitude of the tide is

(1)  $\frac{\pi}{6}$

(2) 4.8

(3) 3

(4) 5.1

3 \_\_\_\_\_

4. In watching auditions for lead singer in a band, Liem became curious as to whether there is an association between how animated the lead singer is and the amount of applause from the audience. He decided to watch each singer and rate the singer on a scale of 1 to 5, where 1 is the least animated and 5 is the most animated. He did this for all 5 nights of auditions and found that the more animated singers did receive louder applause.

The study Liem conducted would be best described as

(1) experimental

(3) a sample survey

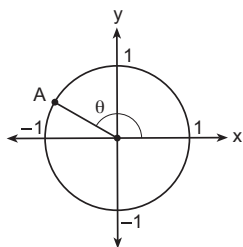
(2) observational

(4) a random assignment

4 \_\_\_\_\_

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5. In the diagram of a unit circle, point  $A$ ,  $\left(-\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$ , represents the point where the terminal side of  $\theta$  intersects the unit circle.



What is  $m\angle\theta$ ?

- (1)  $30^\circ$                       (3)  $135^\circ$   
 (2)  $120^\circ$                     (4)  $150^\circ$

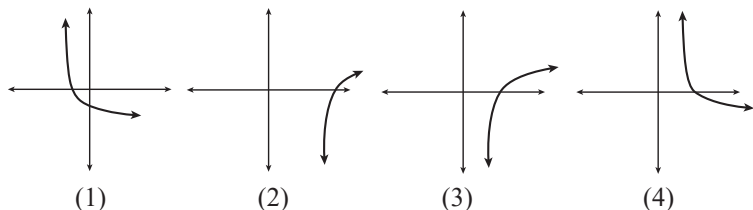
5 \_\_\_\_\_

6. Consider the function  $f(x) = 2x^3 + x^2 - 18x - 9$ . Which statement is true?

- (1)  $2x - 1$  is a factor of  $f(x)$ .                      (3)  $f(3) \neq f\left(-\frac{1}{2}\right)$   
 (2)  $x - 3$  is a factor of  $f(x)$ .                      (4)  $f\left(\frac{1}{2}\right) = 0$

6 \_\_\_\_\_

7. Which sketch could represent the function  $m(x) = -\log_{100}(x - 2)$ ?



7 \_\_\_\_\_

8. Which equation has roots of  $3 + i$  and  $3 - i$ ?

- (1)  $x^2 - 6x + 10 = 0$                       (3)  $x^2 - 10x + 6 = 0$   
 (2)  $x^2 + 6x - 10 = 0$                       (4)  $x^2 + 10x - 6 = 0$

8 \_\_\_\_\_

9. A local university has a current enrollment of 12,000 students. The enrollment is increasing continuously at a rate of 2.5% each year. Which logarithm is equal to the number of years it will take for the population to increase to 15,000 students?

- (1)  $\frac{\ln 1.25}{0.25}$                       (2)  $\frac{\ln 3000}{0.025}$                       (3)  $\frac{\ln 1.25}{2.5}$                       (4)  $\frac{\ln 1.25}{0.025}$

9 \_\_\_\_\_

10. What is the total number of points of intersection of the graphs of the equations  $y = e^x$  and  $xy = 20$ ?

- (1) 1                      (2) 2                      (3) 3                      (4) 0

10 \_\_\_\_\_

11. The amount of a substance,  $A(t)$ , in grams, remaining after  $t$  days is modeled by  $A(t) = 50(0.5)^{\frac{t}{3}}$ . Which statement is false?

- (1) In 20 days, there is no substance remaining.  
 (2) After two half-lives, there is 25% of the substance remaining.  
 (3) The amount of the substance remaining can also be modeled by  $A(t) = 50(2)^{\frac{-t}{3}}$ .  
 (4) After one week, there is less than 10g of the substance remaining.

11 \_\_\_\_\_

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12. A parabola that has a vertex at  $(2, 1)$  and a focus of  $(2, -3)$  has an equation of

(1)  $y = \frac{1}{16}(x - 2)^2 + 1$

(3)  $y = -\frac{1}{16}(x - 2)^2 + 1$

(2)  $y = -\frac{1}{16}(x + 2)^2 - 1$

(4)  $y = -\frac{1}{16}(x - 2)^2 - 3$

12 \_\_\_\_\_

13. The expression  $(a\sqrt[3]{2b^2})\left(\sqrt[3]{4a^2b}\right)$  is equivalent to

(1)  $2ab\sqrt[3]{a^2}$

(2)  $2ab$

(3)  $2ab\sqrt[3]{2a^2}$

(4)  $2a^2b\sqrt[3]{2b}$

13 \_\_\_\_\_

14. Given  $f(x) = 3^{x-1} + 2$ , as  $x \rightarrow -\infty$

(1)  $f(x) \rightarrow -1$

(2)  $f(x) \rightarrow 0$

(3)  $f(x) \rightarrow 2$

(4)  $f(x) \rightarrow -\infty$

14 \_\_\_\_\_

15. For all values of  $x$  for which the expression is defined,  $\frac{x^2 + 3x}{x^2 + 5x + 6}$  is equivalent to

(1)  $1 - \frac{x}{x+2}$

(2)  $\frac{x}{x+2}$

(3)  $\frac{3x}{5x+6}$

(4)  $1 + \frac{1}{2x+6}$

15 \_\_\_\_\_

16. A recursive formula for the sequence 64, 48, 36, ... is

(1)  $a_n = 64(0.75)^{n-1}$

(3)  $a_n = 64 + (n - 1)(-16)$

(2)  $a_1 = 64$

(4)  $a_1 = 64$

$a_n = a_{n-1} - 16$

$a_n = 0.75 a_{n-1}$

16 \_\_\_\_\_

17. Which expression is equivalent to  $\frac{x^3 - 2}{x - 2}$ ?

(1)  $x^2$

(3)  $x^2 - 2$

(2)  $x^2 + 2x + 4 + \frac{6}{x-2}$

(4)  $x^2 - 2x + 4 - \frac{10}{x-2}$

17 \_\_\_\_\_

18. What is the solution set of the equation  $\frac{4}{k^2 - 8k + 12} = \frac{k}{k-2} + \frac{1}{k-6}$ ?

(1)  $\{-1, 6\}$

(2)  $\{1, -6\}$

(3)  $\{-1\}$

(4)  $\{1\}$

18 \_\_\_\_\_

19. Given the polynomial identity  $x^6 + y^6 = (x^2 + y^2)(x^4 - x^2y^2 + y^4)$ , which equation must also be true for all values of  $x$  and  $y$ ?

(1)  $x^6 + y^6 = x^2(x^4 - x^2y^2 + y^4) + y^2(x^4 - x^2y^2 + y^4)$

(2)  $x^6 + y^6 = (x^2 + y^2)(x^2 - y^2)(x^2 - y^2)$

(3)  $(x^3 - y^3)^2 = (x^2 + y^2)(x^4 - x^2y^2 + y^4)$

(4)  $(x^6 + y^6) - (x^2 + y^2) = x^4 - x^2y^2 + y^4$

19 \_\_\_\_\_

20. Given  $p(\theta) = 3\sin\left(\frac{1}{2}\theta\right)$  on the interval  $-\pi < \theta < \pi$ , the function  $p$

(1) decreases, then increases

(3) decreases throughout the interval

(2) increases, then decreases

(4) increases throughout the interval

20 \_\_\_\_\_

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21. A company fired several employees in order to save money. The amount of money the company saved per year over five years following the loss of employees is shown in the table.

Year	Amount Saved (in dollars)
1	59,000
2	64,900
3	71,390
4	78,529
5	86,381.9

Which expression determines the total amount of money saved by the company over 5 years?

(1)  $\frac{59,000 - 59,000(1.1)^5}{1 - 1.1}$

(3)  $\sum_{n=1}^5 59,000(1.1)^n$

(2)  $\frac{59,000 - 59,000(0.1)^5}{1 - 0.1}$

(4)  $\sum_{n=1}^5 59,000(0.1)^{n-1}$

21 \_\_\_\_\_

22. A rush-hour commuter train has arrived on time 64 of its first 80 days. As arrivals continue, which equation can be used to find  $x$ , the number of consecutive days that the train must arrive on schedule to raise its on-time performance rate to 90%?

(1)  $\frac{64}{80+x} = \frac{90}{100}$

(3)  $\frac{64+x}{80} = \frac{90}{100}$

(2)  $\frac{64+x}{80+x} = \frac{90}{100}$

(4)  $\frac{x}{80+x} = \frac{90}{100}$

22 \_\_\_\_\_

23. Given  $f(x) = -\frac{2}{5}x + 4$ , which statement is true of the inverse function  $f^{-1}(x)$ ?

(1)  $f^{-1}(x)$  is a line with slope  $\frac{5}{2}$ . (3)  $f^{-1}(x)$  passes through the point  $(6, -5)$ .

(2)  $f^{-1}(x)$  is a line with slope  $\frac{2}{5}$ . (4)  $f^{-1}(x)$  has a  $y$ -intercept at  $(0, -4)$ . 23 \_\_\_\_\_

24. The amount of a substance,  $A(t)$ , that remains after  $t$  days can be given by the equation  $A(t) = A_0(0.5)^{\frac{t}{0.0803}}$ , where  $A_0$  represents the initial amount of the substance. An equivalent form of this equation is

(1)  $A(t) = A_0(0.000178)^t$

(3)  $A(t) = A_0(0.04015)^t$

(2)  $A(t) = A_0(0.945861)^t$

(4)  $A(t) = A_0(1.08361)^t$

24 \_\_\_\_\_

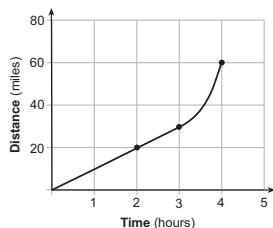


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## Part II

Answer all 8 questions in this part. Each correct answer will receive 2 credits. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. Utilize the information provided for each question to determine your answer. Note that diagrams are not necessarily drawn to scale. For all questions in this part, a correct numerical answer with no work shown will receive only 1 credit. All answers should be written in pen, except for graphs and drawings, which should be done in pencil. [16]

25. Determine the average rate of change, in mph, from 2 to 4 hours on the graph shown.



26. Factor the expression  $x^3 - 2x^2 - 9x + 18$  completely.

27. Solve algebraically for all values of  $x$ :  $\sqrt{4x+1} = 11-x$

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28. Given that  $\left(\frac{y^{\frac{17}{8}}}{y^{\frac{5}{4}}}\right)^{-4} = y^n$ , where  $y > 0$ , determine the value of  $n$ .

29. Given  $\cos A = \frac{3}{\sqrt{10}}$  and  $\cot A = -3$ , determine the value of  $\sin A$  in radical form.

30. According to a study done at a hospital, the average weight of a newborn baby is 3.39 kg, with a standard deviation of 0.55 kg. The weights of all the newborns in this hospital closely follow a normal distribution. Last year, 9256 babies were born at this hospital. Determine, to the *nearest integer*, approximately how many babies weighed more than 4 kg.

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31. The table below shows the results of gender and music preference. Based on these data, determine if the events “the person is female” and “the person prefers classic rock” are independent of each other. Justify your answer.

	Rap	Techno	Classic Rock	Classical
Male	39	17	42	12
Female	17	37	36	15

32. Algebraically determine the solution set for the system of equations below.

$$y = 2x^2 - 7x + 4$$

$$y = 11 - 2x$$

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## Part III

Answer all 4 questions in this part. Each correct answer will receive 4 credits. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. Utilize the information provided for each question to determine your answer. Note that diagrams are not necessarily drawn to scale. For all questions in this part, a correct numerical answer with no work shown will receive only 1 credit. All answers should be written in pen, except for graphs and drawings, which should be done in pencil. [16]

33. When observed by researchers under a microscope, a smartphone screen contained approximately 11,000 bacteria per square inch. Bacteria, under normal conditions, double in population every 20 minutes.

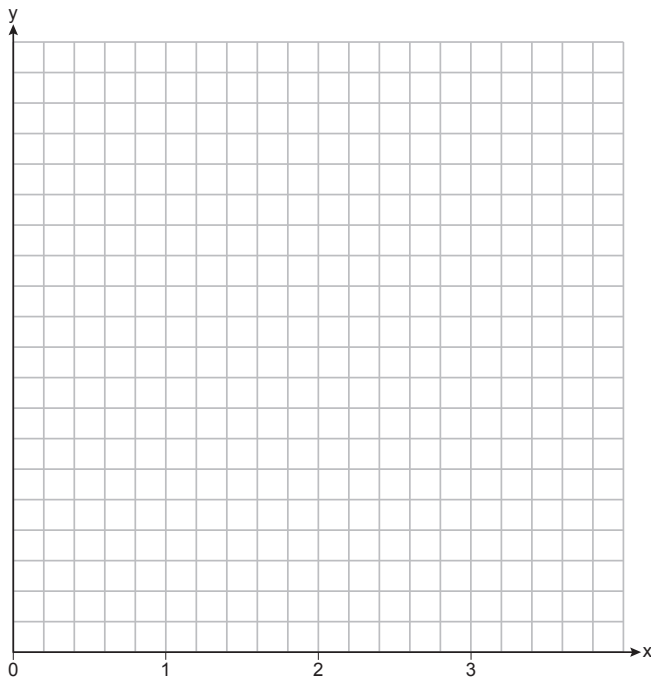
a) Assuming an initial value of 11,000 bacteria, write a function,  $p(t)$ , that can be used to model the population of bacteria,  $p$ , on a smartphone screen, where  $t$  represents the time in minutes after it is first observed under a microscope.

b) Using  $p(t)$  from part a, determine algebraically, to the *nearest hundredth of a minute*, the amount of time it would take for a smartphone screen that was not touched or cleaned to have a population of 1,000,000 bacteria per square inch.

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34. The function  $v(x) = x(3 - x)(x + 4)$  models the volume, in cubic inches, of a rectangular solid for  $0 \leq x \leq 3$ .

Graph  $y = v(x)$  over the domain  $0 \leq x \leq 3$ .



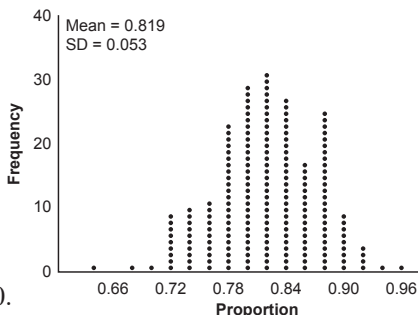
To the *nearest tenth of a cubic inch*, what is the maximum volume of the rectangular solid?

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35. Given  $f(x) = 3x^3 - 4x^2 + 2x - 1$  and  $g(x) = x - 4$ , state the quotient and remainder of  $\frac{f(x)}{g(x)}$ , in the form  $q(x) + \frac{r(x)}{g(x)}$ .

Is  $x = 4$  a root of  $f(x)$ ? Explain your answer.

36. State officials claim 82% of a community want to repeal the 30 mph speed limit on an expressway. A community organization devises a simulation based on the claim that 82% of the community supports the repeal. Each dot on the graph represents the proportion of community members who support the repeal. The graph shows 200 simulated surveys, each of sample size 60.



Based on the simulation, determine an interval containing the middle 95% of plausible proportions. Round your answer to the *nearest thousandth*.

The community organization conducted its own sample survey of 60 people and found 70% supported the repeal. Based on the results of the simulation, explain why the organization should question the State officials' claim.

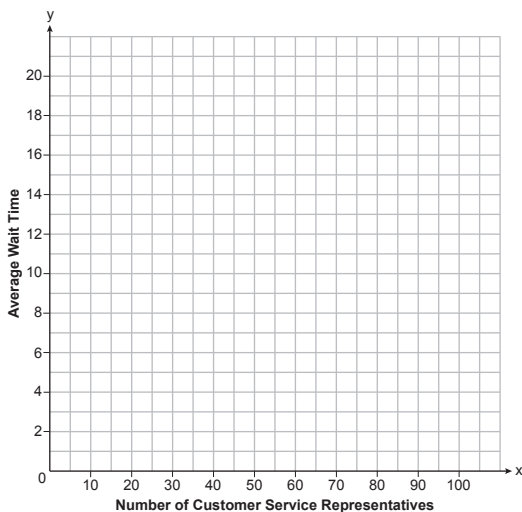
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## Part IV

Answer the question in this part. A correct answer will receive 6 credits. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. Utilize the information provided to determine your answer. Note that diagrams are not necessarily drawn to scale. A correct numerical answer with no work shown will receive only 1 credit. All answers should be written in pen, except for graphs and drawings, which should be done in pencil. [6]

37. A technology company is comparing two plans for speeding up its technical support time. Plan  $A$  can be modeled by the function  $A(x) = 15.7(0.98)^x$  and plan  $B$  can be modeled by the function  $B(x) = 11(0.99)^x$  where  $x$  is the number of customer service representatives employed by the company and  $A(x)$  and  $B(x)$  represent the average wait time, in minutes, of each customer.

Graph  $A(x)$  and  $B(x)$  in the interval  $0 \leq x \leq 100$  on the set of axes.



To the *nearest integer*, solve the equation  $A(x) = B(x)$ .

Determine, to the *nearest minute*,  $B(100) - A(100)$ . Explain what this value represents in the given context.

January 2023

## Part I

Answer all 24 questions in this part. Each correct answer will receive 2 credits. No partial credit will be allowed. Utilize the information provided for each question to determine your answer. Note that diagrams are not necessarily drawn to scale. For each statement or question, choose the word or expression that, of those given, best completes the statement or answers the question. Record your answers in the space provided. [48]

1. Which expression is equivalent to  $(x + 2)^2 - 5(x + 2) + 6$ ?

- (1)  $x(x - 1)$  (3)  $(x - 4)(x + 3)$   
 (2)  $(x - 3)(x - 2)$  (4)  $(x - 6)(x + 1)$  1 \_\_\_\_\_

2. To the nearest tenth, the solution to the equation

$$4300e^{0.07x} - 123 = 5000$$
 is

- (1) 1.1 (2) 2.5 (3) 6.3 (4) 68.5 2 \_\_\_\_\_

3. The value of an automobile  $t$  years after it was purchased is given by the function  $V = 38,000(0.84)^t$ . Which statement is true?

- (1) The value of the car increases 84% each year.  
 (2) The value of the car decreases 84% each year.  
 (3) The value of the car increases 16% each year.  
 (4) The value of the car decreases 16% each year. 3 \_\_\_\_\_

4. Which function represents exponential decay?

- (1)  $p(x) = \left(\frac{1}{4}\right)^{-x}$  (2)  $q(x) = 1.8^{-x}$  (3)  $r(x) = 2.3^{2x}$  (4)  $s(x) = 4^{\frac{x}{2}}$  4 \_\_\_\_\_

5. The expression  $\frac{x^4 - 5x^2 + 4x + 14}{x + 2}$  is equivalent to

- (1)  $x^3 - 2x^2 - x + 6 + \frac{2}{x + 2}$  (3)  $x^3 + 2x^2 - x + 2 + \frac{18}{x + 2}$   
 (2)  $x^3 - 5x + 4 - \frac{14}{x + 2}$  (4)  $x^3 + 2x^2 - 9x + 22 - \frac{30}{x + 2}$  5 \_\_\_\_\_

6. The sum of the first 20 terms of the series  $-2 + 6 - 18 + 54 - \dots$  is

- (1) -610 (2) -59 (3) 1,743,392,200 (4) 2,324,522,934 6 \_\_\_\_\_

7. If  $f(x) = 2x^4 - x^3 - 16x + 8$ , then  $f\left(\frac{1}{2}\right)$

- (1) equals 0 and  $2x + 1$  is a factor of  $f(x)$   
 (2) equals 0 and  $2x - 1$  is a factor of  $f(x)$   
 (3) does not equal 0 and  $2x + 1$  is not a factor of  $f(x)$   
 (4) does not equal 0 and  $2x - 1$  is a factor of  $f(x)$  7 \_\_\_\_\_

8. If  $(6 - ki)^2 = 27 - 36i$ , the value of  $k$  is

- (1) -36 (2) -3 (3) 3 (4) 6 8 \_\_\_\_\_



9. What is the solution set of the equation  $\frac{x+2}{x} + \frac{x}{3} = \frac{2x^2+6}{3x}$ ?

- (1)  $\{-3\}$       (2)  $\{-3, 0\}$       (3)  $\{3\}$       (4)  $\{0, 3\}$       9 \_\_\_\_\_

10. How many real solutions exist for the system of equations below?

$$y = \frac{1}{4}x - 8$$

$$y = \frac{1}{2}x^2 + 2x$$

- (1) 1      (2) 2      (3) 3      (4) 0      10 \_\_\_\_\_

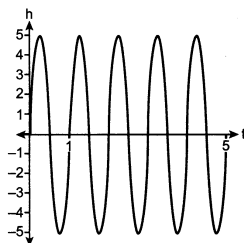
11. Which equation represents a polynomial identity?

- (1)  $x^3 + y^3 = (x + y)^3$       (3)  $x^3 + y^3 = (x + y)(x^2 - xy + y^2)$   
 (2)  $x^3 + y^3 = (x + y)(x^2 - xy + y^2)$       (4)  $x^3 + y^3 = (x - y)(x^2 + xy + y^2)$       11 \_\_\_\_\_

12. Given  $x > 0$ , the expression  $\frac{x^{\frac{5}{2}}}{x^{\frac{1}{2}}}$  can be rewritten as

- (1)  $\sqrt[3]{x}$       (2)  $-\sqrt[10]{x^3}$       (3)  $\frac{1}{\sqrt[10]{x^3}}$       (4)  $\sqrt[3]{x^{10}}$       12 \_\_\_\_\_

13. A cyclist pedals a bike at a rate of 60 revolutions per minute. The height,  $h$ , of a pedal at time  $t$ , in seconds, is plotted to the right.



The graph can be modeled by the function

- $h(t) = 5\sin(kt)$ , where  $k$  is equal to  
 (1) 1      (2)  $2\pi$       (3) 60      (4)  $\frac{\pi}{30}$       13 \_\_\_\_\_

14. Which statement about data collection is most accurate?

- (1) A survey about parenting styles given to every tenth student entering the library will provide unbiased results.  
 (2) An observational study allows a researcher to determine the cause of an outcome.  
 (3) Margin of error increases as sample size increases.  
 (4) A survey collected from a random sample of students in a school can be used to represent the opinions of the school population.      14 \_\_\_\_\_

15. If  $f(x) = \frac{1}{2}x + 2$ , then the inverse function is

- (1)  $f^{-1}(x) = -\frac{1}{2}x - 2$       (3)  $f^{-1}(x) = 2x - 4$   
 (2)  $f^{-1}(x) = \frac{1}{2}x - 1$       (4)  $f^{-1}(x) = 2x + 2$       15 \_\_\_\_\_

16. Given  $f(x) = x^4 - x^3 - 6x^2$ , for what values of  $x$  will  $f(x) > 0$ ?

- (1)  $x < -2$ , only      (3)  $x < -2$  or  $0 \leq x \leq 3$   
 (2)  $x < -2$  or  $x > 3$       (4)  $x > 3$ , only      16 \_\_\_\_\_

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17. For which approximate value(s) of  $x$  will  $\log(x + 5) = |x - 1| - 3$ ?

- (1) 5, 1      (2) -2.41, 0.41      (3) -2.41, 5      (4) 5, only      17 \_\_\_\_\_

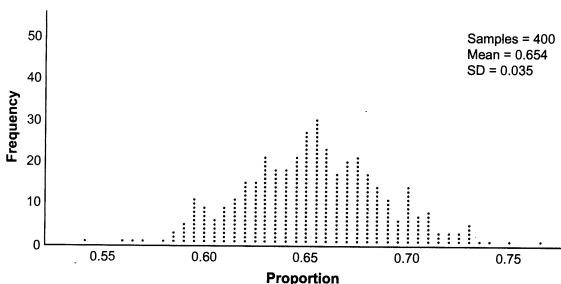
18. Consider a cubic polynomial with the characteristics below.

- exactly one real root
- as  $x \rightarrow \infty$ ,  $f(x) \rightarrow -\infty$

Given  $a > 0$  and  $b > 0$ , which equation represents a cubic polynomial with these characteristics?

- (1)  $f(x) = (x - a)(x^2 + b)$       (3)  $f(x) = (a - x^2)(x^2 + b)$   
 (2)  $f(x) = (a - x)(x^2 + b)$       (4)  $f(x) = (x - a)(b - x^2)$       18 \_\_\_\_\_

19. Betty conducted a survey of her class to see if they like pizza. She gathered 200 responses and 65% of the voters said they did like pizza. Betty then ran a simulation of 400 more surveys, each with 200 responses, assuming that 65% of the voters would like pizza. The output of the simulation is shown below.



Considering the middle 95% of the data, what is the margin of error for the simulation?

- (1) 0.01      (2) 0.02      (3) 0.05      (4) 0.07      19 \_\_\_\_\_

20. If  $\cos A = \frac{\sqrt{5}}{3}$  and  $\tan A < 0$ , what is the value of  $\sin A$ ?

- (1)  $\frac{2}{3}$       (2)  $-\frac{\sqrt{5}}{3}$       (3)  $-\frac{2}{3}$       (4)  $\frac{3}{\sqrt{5}}$       20 \_\_\_\_\_

21. A tree farm initially has 150 trees. Each year, 20% of the trees are cut down and 80 seedlings are planted. Which recursive formula models the number of trees,  $a_n$ , after  $n$  years?

- (1)  $a_1 = 150$       (3)  $a_n = 150(0.2)^n + 80$   
 $a_n = a_{n-1}(0.2) + 80$   
 (2)  $a_1 = 150$       (4)  $a_n = 150(0.8)^n + 80$   
 $a_n = a_{n-1}(0.8) + 80$       21 \_\_\_\_\_

22. Which equation represents a parabola with a focus of  $(4, -3)$  and directrix of  $y = 1$ ?

- (1)  $(x - 1)^2 = 4(y + 3)$       (3)  $(x + 4)^2 = 4(y - 3)$   
 (2)  $(x - 1)^2 = -8(y - 3)$       (4)  $(x - 4)^2 = -8(y + 1)$       22 \_\_\_\_\_

January 2023

23. Mia has a student loan that is in deferment, meaning that she does not need to make payments right now. The balance of her loan account during her deferment can be represented by the function  $f(x) = 35,000(1.0325)^x$ , where  $x$  is the number of years since the deferment began. If the bank decides to calculate her balance showing a monthly growth rate, an approximately equivalent function would be

(1)  $f(x) = 35,000(1.0027)^{12x}$

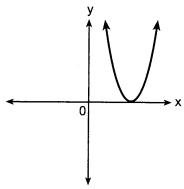
(3)  $f(x) = 35,000(1.0325)^{12x}$

(2)  $f(x) = 35,000(1.0027)^{\frac{x}{12}}$

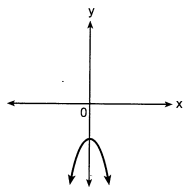
(4)  $f(x) = 35,000(1.0325)^{\frac{x}{12}}$

23 \_\_\_\_\_

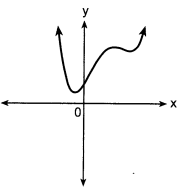
24. Which graph shows a quadratic function with two imaginary zeros?



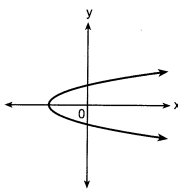
(1)



(2)



(3)



(4)

24 \_\_\_\_\_

### Part II

Answer all 8 questions in this part. Each correct answer will receive 2 credits. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. Utilize the information provided for each question to determine your answer. Note that diagrams are not necessarily drawn to scale. For all questions in this part, a correct numerical answer with no work shown will receive only 1 credit. All answers should be written in pen, except for graphs and drawings, which should be done in pencil. [16]

25. Algebraically determine the zeros of the function below.

$$r(x) = 3x^3 + 12x^2 - 3x - 12$$

26. Given  $a > 0$ , solve the equation  $a^{x+1} = \sqrt[3]{a^2}$  for  $x$  algebraically.

27. Given  $P(A) = \frac{1}{3}$  and  $P(B) = \frac{5}{12}$ , where  $A$  and  $B$  are independent events, determine  $P(A \cap B)$ .

28. The scores on a collegiate mathematics readiness assessment are approximately normally distributed with a mean of 680 and a standard deviation of 120.

Determine the percentage of scores between 690 and 900, to the *nearest percent*.

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29. Consider the data in the accompanying table.

<b>x</b>	1	2	3	4	5	6
<b>y</b>	3.9	6	11	18.1	28	40.3

State an exponential regression equation to model these data, rounding all values to the *nearest thousandth*.

30. Write the expression  $A(x) \cdot B(x) - 3C(x)$  as a polynomial in standard form.

$$A(x) = x^3 + 2x - 1$$

$$B(x) = x^2 + 7$$

$$C(x) = x^4 - 5x$$

31. Over the set of integers, completely factor  $x^4 - 5x^2 + 4$ .

## January 2023

32. Natalia's teacher has given her the following information about angle  $\theta$ .

- $\pi < \theta < 2\pi$
- $\cos \theta = \frac{\sqrt{3}}{4}$

Explain how Natalia can determine if the value of  $\tan \theta$  is positive or negative.

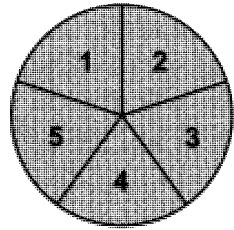
**Part III**

**Answer all 4 questions in this part. Each correct answer will receive 4 credits. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. Utilize the information provided for each question to determine your answer. Note that diagrams are not necessarily drawn to scale. For all questions in this part, a correct numerical answer with no work shown will receive only 1 credit. All answers should be written in pen, except for graphs and drawings, which should be done in pencil. [16]**

33. Solve the equation  $\sqrt{49 - 10x} + 5 = 2x$  algebraically.

January 2023

34. Joette is playing a carnival game. To win a prize, one has to correctly guess which of five equally sized regions a spinner will land on, as shown in the diagram.

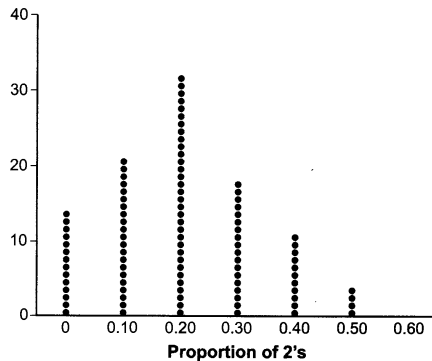


She complains that the game is unfair because her favorite number, 2, has only been spun once in ten times she played the game.

State the proportion of 2's that were spun.

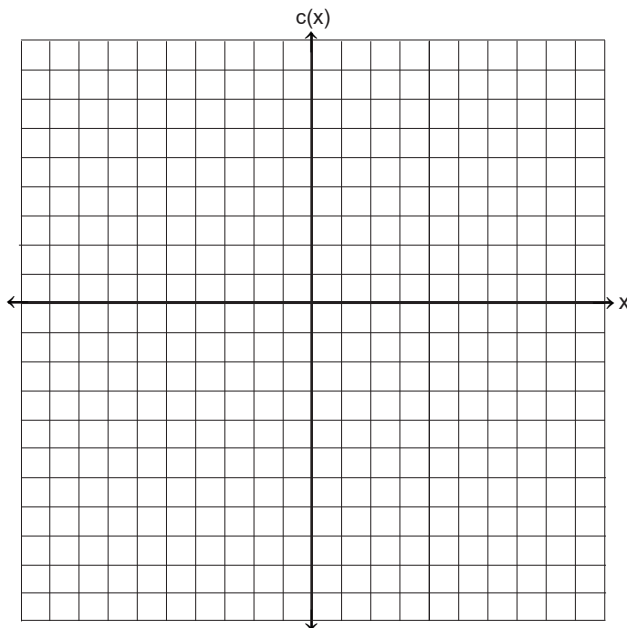
State the theoretical probability of spinning a 2.

The simulation output shows the results of simulating ten spins of a fair spinner, repeated 100 times.



Does the output indicate that the carnival game was unfair? Explain your answer.

January 2023

35. Graph  $c(x) = -9(3)^{x-4} + 2$  on the axes below.Describe the end behavior of  $c(x)$  as  $x$  approaches positive infinity.Describe the end behavior of  $c(x)$  as  $x$  approaches negative infinity.36. The monthly high temperature ( $^{\circ}\text{F}$ ) in Buffalo, New York can be modeled by  $B(m) = 24.9\sin(0.5m - 2.05) + 55.25$ , where  $m$  is the number of the month and January = 1.Find the average rate of change in the monthly high temperature between June and October, to the *nearest hundredth*.

Explain what this value represents in the given context.



January 2023

## Part IV

Answer the question in this part. A correct answer will receive 6 credits. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. Utilize the information provided to determine your answer. Note that diagrams are not necessarily drawn to scale. A correct numerical answer with no work shown will receive only 1 credit. All answers should be written in pen, except for graphs and drawings, which should be done in pencil. [6]

37. Objects cool at different rates based on the formula below.

$$T = (T_0 - T_R)e^{-rt} + T_R$$

$T_0$ : initial temperature

$T_R$ : room temperature

$r$ : rate of cooling of the object

$t$ : time in minutes that the object cools to a temperature,  $T$

Mark makes T-shirts using a hot press to transfer designs to the shirts. He removes a shirt from a press that heats the shirt to 400°F. The rate of cooling for the shirt is 0.0735 and the room temperature is 75°F. Using this information, write an equation for the temperature of the shirt,  $T$ , after  $t$  minutes.

Use the equation to find the temperature of the shirt, to the *nearest degree*, after five minutes.

Question 37 is continued on the next page.

**ALGEBRA 2 – COMMON CORE**  
**January 2023****Question 37 is continued**

At the same time, Mark's friend Jeanine removes a hoodie from a press that heats the hoodie to  $450^{\circ}\text{F}$ . After eight minutes, the hoodie measured  $270^{\circ}\text{F}$ . The room temperature is still  $75^{\circ}\text{F}$ . Determine the rate of cooling of the hoodie, to the *nearest ten thousandth*.

The T-shirt and hoodie were removed at the same time. Determine when the temperature will be the same, to the *nearest minute*.

June 2023

## Part I

Answer all 24 questions in this part. Each correct answer will receive 2 credits. No partial credit will be allowed. Utilize the information provided for each question to determine your answer. Note that diagrams are not necessarily drawn to scale. For each statement or question, choose the word or expression that, of those given, best completes the statement or answers the question. Record your answers in the space provided. [48]

1. The population of Austin, Texas from 1850 to 2010 is summarized in the table below.

Year	1850	1870	1890	1910	1930	1950	1970	1990	2010
Population	629	4428	14,575	29,860	53,120	132,459	251,808	494,290	790,390

Over which period of time was the average rate of change in population the greatest?

- (1) 1850 to 1910 (3) 1950 to 1970  
 (2) 1990 to 2010 (4) 1890 to 1970 1 \_\_\_\_\_

2. Which expression is *not* equivalent to  $36x^6 - 25y^4$ ?

- (1)  $6^2(x^3)^2 - 5^2(y^2)^2$  (3)  $(6x^6 - 5y^4)(6x^6 + 5y^4)$   
 (2)  $(6x^3 - 5y^2)(6x^3 + 5y^2)$  (4)  $(3 \cdot 2x^3 - 5y^2)(3 \cdot 2x^3 + 5y^2)$  2 \_\_\_\_\_

3. What are the zeros of  $s(x) = x^4 - 9x^2 + 3x^3 - 27x - 10x^2 + 90$ ?

- (1)  $\{-3, -2, 5\}$  (2)  $\{-2, 3, 5\}$  (3)  $\{-3, -2, 3, 5\}$  (4)  $\{-5, -3, 2, 3\}$  3 \_\_\_\_\_

4. If  $\theta$  is an angle in standard position whose terminal side passes through the point  $(-2, -3)$ , what is the numerical value of  $\tan \theta$ ?

- (1)  $\frac{2}{3}$  (2)  $\frac{3}{2}$  (3)  $-\frac{2}{\sqrt{13}}$  (4)  $-\frac{3}{\sqrt{13}}$  4 \_\_\_\_\_

5. The average monthly temperature,  $T(m)$ , in degrees Fahrenheit, over a 12 month period, can be modeled by  $T(m) = -23 \cos\left(\frac{\pi}{6} m\right) + 56$ , where  $m$  is in months. What is the range of temperatures, in degrees Fahrenheit, of this function?

- (1)  $[-23, 23]$  (2)  $[33, 79]$  (3)  $[-23, 56]$  (4)  $[-79, 33]$  5 \_\_\_\_\_

6. Which expression is an equivalent form of  $a^5\sqrt{a^4}$ ?

- (1)  $a$  (2)  $a^{\frac{9}{5}}$  (3)  $a^{\frac{9}{4}}$  (4)  $a^{\frac{1}{5}}$  6 \_\_\_\_\_

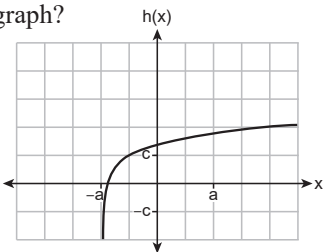
7. The expression  $3i(ai - 6i^2)$  is equivalent to

- (1)  $3a + 18i$  (2)  $3a - 18i$  (3)  $-3a + 18i$  (4)  $-3a - 18i$  7 \_\_\_\_\_

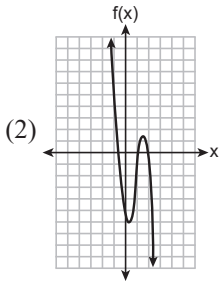
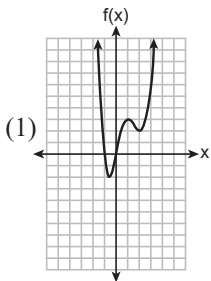
June 2023

8. Which equation best represents the graph?

- (1)  $h(x) = \log(x + a) + c$   
 (2)  $h(x) = \log(x - a) + c$   
 (3)  $h(x) = \log(x + a) - c$   
 (4)  $h(x) = \log(x - a) - c$



8 \_\_\_\_\_

9. Which function has the characteristic as  $x \rightarrow -\infty, f(x) \rightarrow -\infty$ ?

- (3)  $f(x) = 5(4)^{-x}$  (4)  $f(x) = -\log_5(-x)$

9 \_\_\_\_\_

10. The expression  $(x^2 + 3)^2 - 2(x^2 + 3) - 24$  is equivalent to

- (1)  $(x^2 + 9)(x^2 - 1)$  (3)  $x^4 - 2x^2 - 21$   
 (2)  $(x^2 - 3)(x^2 + 7)$  (4)  $x^4 + 4x^2 - 9$

10 \_\_\_\_\_

11. What is the solution for the system of equations below?

$$\begin{aligned} x + y + z &= 2 \\ x - 2y - z &= -4 \\ x - 9y + z &= -18 \end{aligned}$$

- (1)  $(-2, 2, 2)$  (2)  $(-2, -2, 6)$  (3)  $(0, 2, 0)$  (4)  $(0, 2, 4)$  11 \_\_\_\_\_

12. The roots of the equation  $x^2 - 4x = -13$  are

- (1)  $2 \pm 3i$  (2)  $2 \pm 6i$  (3)  $2 \pm \sqrt{17}$  (4)  $2 \pm i\sqrt{13}$  12 \_\_\_\_\_

13. Which expression is equivalent to  $\frac{2x^3 + 2x - 7}{2x + 4}$ ?

- (1)  $x^2 - 2x + 5 - \frac{27}{2x + 4}$  (3)  $x^2 + 2x + 5 + \frac{13}{2x + 4}$

- (2)  $x^2 - 1 - \frac{3}{2x + 4}$  (4)  $x^2 + 2x - 3 + \frac{5}{2x + 4}$  13 \_\_\_\_\_

**June 2023**

14. A popular celebrity tracks the number of people, in thousands, who have followed her on social media since January 1, 2015. A summary of the data she recorded is shown in the table below:

<b>Number of Months Since January 2015</b>	2	11	16	20	27	35	47	50	52
<b>Number of Social Media Followers (thousands)</b>	3.1	7.5	29.7	49.7	200.3	680.3	5200.3	8109.3	12,107.1

The celebrity uses an exponential regression equation to model the data. According to the model, about how many followers did she have on June 1, 2018?

- (1) 13,000,000      (2) 5,420,000      (3) 1,850,000      (4) 790,000      14 \_\_\_\_\_

15. Luminescence is the emission of light that is not caused by heat. A luminescent substance decays according to the function below.

$$I = I_0 e^{3\left(-\frac{t}{0.6}\right)}$$

This function can be best approximated by

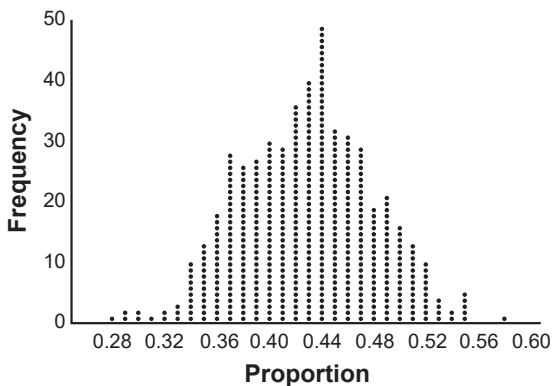
- (1)  $I = I_0 e^{\left(-\frac{t}{0.18}\right)}$       (3)  $I = I_0 (0.0067)^t$   
 (2)  $I = I_0 e^{5t}$       (4)  $I = I_0 (0.0497)^{0.6t}$       15 \_\_\_\_\_

16. The heights of the students at Central High School can be modeled by a normal distribution with a mean of 68.1 and a standard deviation of 3.4 inches. According to this model, approximately what percent of the students would have a height less than 60 inches or greater than 75 inches?

- (1) 0.86%      (2) 1.26%      (3) 2.12%      (4) 2.98%      16 \_\_\_\_\_

17. Marissa and Sydney are trying to determine if there is enough interest in their school to put on a senior musical. They randomly surveyed

100 members of the senior class and 43% of them said they would be interested in being in a senior musical. Marissa and Sydney then conducted a simulation of 500 more surveys, each of 100 seniors, assuming that 43% of the senior class would be interested in being in the musical. The output of the simulation is shown.



The standard deviation of the simulation is closest to

- (1) 0.02      (2) 0.05      (3) 0.09      (4) 0.43      17 \_\_\_\_\_

June 2023

18. For  $f(x) = \cos x$ , which statement is true?

- (1)  $2f(x)$  and  $f(2x)$  are even functions.  
 (2)  $f(2x)$  and  $f(x) + 2$  are odd functions.  
 (3)  $2f(x)$  and  $f\left(x + \frac{\pi}{2}\right)$  are odd functions.  
 (4)  $f(x) + 2$  is an odd function and  $f\left(x + \frac{\pi}{2}\right)$  is an even function. 18 \_\_\_\_\_

19. The solution set of  $\frac{x+3}{x-5} + \frac{6}{x+2} = \frac{6+10x}{(x-5)(x+2)}$  is

- (1)  $\{-6\}$       (2)  $\{5\}$       (3)  $\{-6, 5\}$       (4)  $\{-5, 6\}$  19 \_\_\_\_\_

20. Given  $x$  and  $y$  are positive, which expressions are equivalent to  $\frac{x^3}{y}$ ?

I.  $\left(\frac{y}{x^3}\right)^{-1}$

II.  $\sqrt[3]{x^9}(y^{-1})$

III.  $\frac{x^6\sqrt[4]{y^8}}{x^3y^3}$

- (1) I and II, only      (3) II and III, only  
 (2) I and III, only      (4) I, II, and III 20 \_\_\_\_\_

21. Given the inverse function  $f^{-1}(x) = \frac{2}{3}x + \frac{1}{6}$ , which function represents  $f(x)$ ?

- (1)  $f(x) = -\frac{2}{3}x + \frac{1}{6}$       (3)  $f(x) = \frac{3}{2}x - \frac{1}{4}$   
 (2)  $f(x) = -\frac{3}{2}x + \frac{1}{4}$       (4)  $f(x) = \frac{3}{2}x - \frac{1}{6}$  21 \_\_\_\_\_

22. How many equations below are identities?

•  $x^2 + y^2 = (x^2 - y^2) + (2xy)^2$

•  $x^3 + y^3 = (x - y) + (x^2 - xy + y^2)$

•  $x^4 + y^4 = (x - y)(x - y)(x^2 + y^2)$

- (1) 1      (2) 2      (3) 3      (4) 0 22 \_\_\_\_\_

23. If the focus of a parabola is  $(0, 6)$  and the directrix is  $y = 4$ , what is an equation for the parabola?

- (1)  $y^2 = 4(x - 5)$       (3)  $y^2 = 8(x - 5)$   
 (2)  $x^2 = 4(y - 5)$       (4)  $x^2 = 8(y - 6)$  23 \_\_\_\_\_

24. John and Margaret deposit \$500 into a savings account for their son on his first birthday. They continue to make a deposit of \$500 on the child's birthday, with the last deposit being made on the child's 21st birthday. If the account pays 4% annual interest, which equation represents the amount of money in the account after the last deposit is made?

- (1)  $S_{21} = 500(1.04)^{21}$       (3)  $S_{21} = 500(1.04)^{20} + 500$   
 (2)  $S_{21} = \frac{500(1 - 1.04^{21})}{1 - 1.04}$       (4)  $S_{21} = \frac{500(1 - 0.04^{21})}{1 - 1.04}$  24 \_\_\_\_\_

June 2023

## Part II

Answer all 8 questions in this part. Each correct answer will receive 2 credits. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. Utilize the information provided for each question to determine your answer. Note that diagrams are not necessarily drawn to scale. For all questions in this part, a correct numerical answer with no work shown will receive only 1 credit. All answers should be written in pen, except for graphs and drawings, which should be done in pencil. [16]

25. The business office of a local college wishes to determine the methods of payment that will be used by students when buying books at the beginning of a semester. Explain how the office can gather an appropriate sample that minimizes bias.

26. Determine the solution of  $\sqrt{3x+7} = x-1$  algebraically.

June 2023

27. The population of bacteria,  $P(t)$ , in hundreds, after  $t$  hours can be modeled by the function  $P(t) = 37e^{0.0532t}$ . Determine whether the population is increasing or decreasing over time. Explain your reasoning.

28. The polynomial function  $g(x) = x^3 + ax^2 - 5x + 6$  has a factor of  $(x - 3)$ . Determine the value of  $a$ .

29. Write a recursive formula for the sequence 189, 63, 21, 7, ... .



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30. Solve algebraically for  $x$  to the *nearest thousandth*:

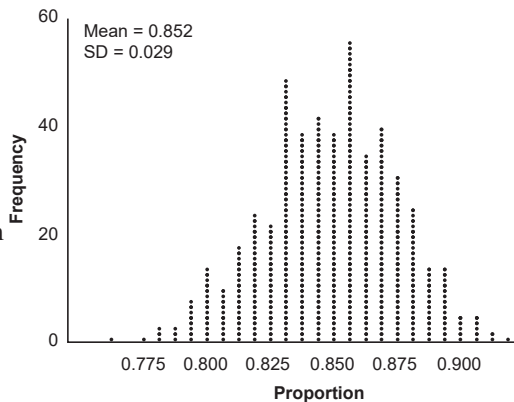
$$2e^{0.49x} = 15$$

31. For all values of  $x$  for which the expression is defined, write the expression below in simplest form.

$$\frac{2x^3 + x^2 - 18x - 9}{3x - x^2}$$

June 2023

32. An app design company believes that the proportion of high school students who have purchased apps on their smartphones in the past 3 months is 0.85. A simulation of 500 samples of 150 students was run based on this proportion and the results are shown.



Suppose a sample of 150 students from your high school showed that 88% of students had purchased apps on their smartphones in the past 3 months. Based on the simulation, would the results from your high school give the app design company reason to believe their assumption is *incorrect*? Explain.

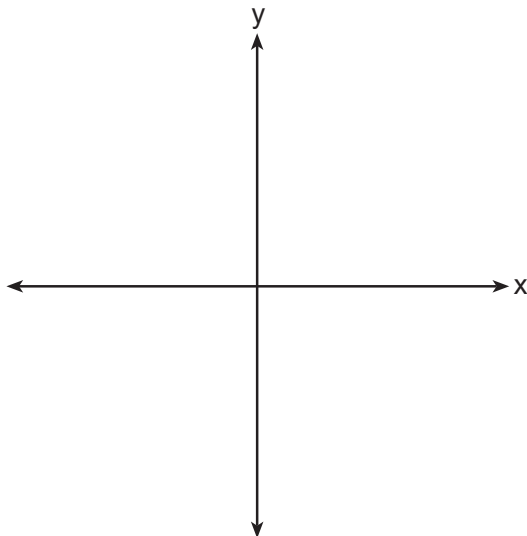
June 2023

## Part III

Answer all 4 questions in this part. Each correct answer will receive 4 credits. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. Utilize the information provided for each question to determine your answer. Note that diagrams are not necessarily drawn to scale. For all questions in this part, a correct numerical answer with no work shown will receive only 1 credit. All answers should be written in pen, except for graphs and drawings, which should be done in pencil. [16]

33. Patricia creates a cubic polynomial function,  $p(x)$ , with a leading coefficient of 1. The zeros of the function are 2, 3, and  $-6$ . Write an equation for  $p(x)$ .

Sketch  $y = p(x)$  on the set of axes below.



June 2023

34. A public radio station held a fund-raiser. The table below summarizes the donor category and method of donation.

		Donor Category	
		Supporter	Patron
Method of Donation	Phone calls	400	672
	Online	1200	2016

To the *nearest thousandth*, find the probability that a randomly selected donor was categorized as a supporter, given that the donation was made online.

Do these data indicate that being a supporter is independent of donating online? Justify your answer.

35. Algebraically solve the system:

$$\begin{aligned}(x - 2)^2 + (y - 3)^2 &= 20 \\ y &= -2x + 7\end{aligned}$$

## June 2023

36. On a certain tropical island, there are currently 500 palm trees and 200 flamingos. Suppose the palm tree population is decreasing at an annual rate of 3% per year and the flamingo population is growing at a continuous rate of 2% per year.

Write two functions,  $P(x)$  and  $F(x)$ , that represent the number of palm trees and flamingos on this island, respectively,  $x$  years from now.

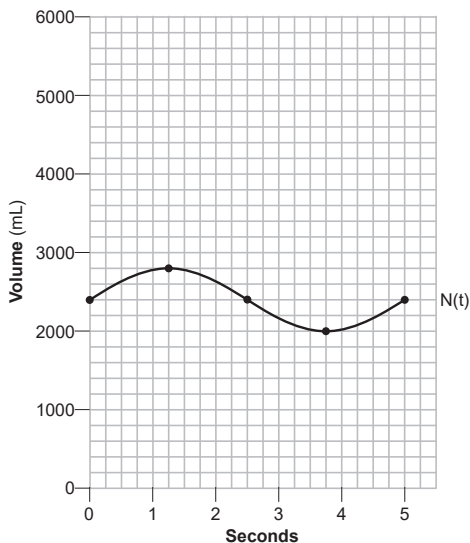
State the solution to the equation  $P(x) = F(x)$ , rounded to the *nearest year*. Interpret the meaning of this value within the given context.

June 2023

## Part IV

Answer the question in this part. A correct answer will receive 6 credits. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. Utilize the information provided to determine your answer. Note that diagrams are not necessarily drawn to scale. A correct numerical answer with no work shown will receive only 1 credit. All answers should be written in pen, except for graphs and drawings, which should be done in pencil. [6]

37. The volume of air in an average lung during breathing can be modeled by the graph.



Using the graph, write an equation for  $N(t)$ , in the form  $N(t) = A \sin(Bt) + C$ .

That same lung, when engaged in exercise, has a volume that can be modeled by  $E(t) = 2000 \sin(\pi t) + 3200$ , where  $E(t)$  is volume in mL and  $t$  is time in seconds.

Graph *at least one* cycle of  $E(t)$  on the same grid as  $N(t)$ .

How many times during the 5-second interval will  $N(t) = E(t)$ ?

August 2023

## Part I

Answer all 24 questions in this part. Each correct answer will receive 2 credits. No partial credit will be allowed. Utilize the information provided for each question to determine your answer. Note that diagrams are not necessarily drawn to scale. For each statement or question, choose the word or expression that, of those given, best completes the statement or answers the question. Record your answers in the space provided. [48]

1. A group of high school students wanted to collect information on how many times per week students exercised. If they want the *least* biased results they should survey every fifth student at the school who is

- (1) entering the gym (3) entering the library  
(2) in the junior class (4) entering the building 1 \_\_\_\_\_

2. Given  $x \neq -3$ , which expression is equivalent to  $\frac{2x^3 + 3x^2 - 4x + 5}{x + 3}$ ?

- (1)  $2x^3 + 9x^2 + 23x + 74$  (3)  $2x^3 - 3x^2 + 5x - 10$   
(2)  $2x^2 - 3x + 5 - \frac{10}{x + 3}$  (4)  $2x^2 + 9x + 23 + \frac{74}{x + 3}$  2 \_\_\_\_\_

3. The table shows the food preferences of sports fans whose favorite sport is football or baseball.

Favorite Food to Eat While Watching Sports

	Wings	Pizza	Hot Dogs
Football	14	20	6
Baseball	6	12	42

The probability that a fan prefers pizza given that the fan prefers football is

- (1)  $\frac{1}{2}$  (2)  $\frac{1}{5}$  (3)  $\frac{5}{8}$  (4)  $\frac{13}{25}$  3 \_\_\_\_\_

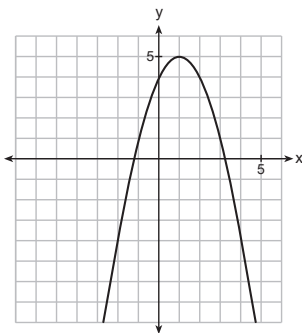
4. If  $f(x) = 12x - 4$ , then the inverse function  $f^{-1}(x)$  is

- (1)  $f^{-1}(x) = \frac{x + 1}{3}$  (3)  $f^{-1}(x) = \frac{x + 4}{12}$   
(2)  $f^{-1}(x) = \frac{x}{3} + 1$  (4)  $f^{-1}(x) = \frac{x}{12} + 4$  4 \_\_\_\_\_

5. The graph of a quadratic function is shown.

When the graph of  $x + y = 4$  is drawn on the same axes, one solution to this system is

- (1) (4, 0) (3) (2, 2)  
(2) (1, 5) (4) (3, 1)



5 \_\_\_\_\_

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6. What is the solution of  $2(3^{x+4}) = 56$ ?

(1)  $x = \log_3(28) - 4$

(3)  $x = \log(25) - 4$

(2)  $x = -1$

(4)  $x = \frac{\log(56)}{\log(6)} - 4$

6 \_\_\_\_\_

7. In a survey of people who recently bought a laptop, 45% said they were looking for a large screen, 31% said they were looking for a fast processor, and 58% said they wanted a large screen or a fast processor. If a survey respondent is selected at random, what is the probability that the respondent wanted both a large screen and a fast processor?

(1) 76%

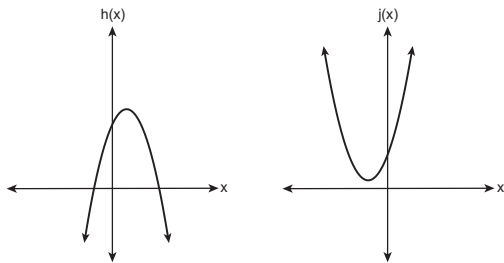
(2) 14%

(3) 77%

(4) 18%

7 \_\_\_\_\_

8. In the quadratic formula,  $b^2 - 4ac$  is called the discriminant. The function  $f(x)$  has a discriminant value of 8, and  $g(x)$  has a discriminant value of  $-16$ . The quadratic graphs,  $h(x)$  and  $j(x)$ , are shown below.



Which quadratic functions have imaginary roots?

(1)  $g(x)$  and  $h(x)$

(3)  $f(x)$  and  $h(x)$

(2)  $g(x)$  and  $j(x)$

(4)  $f(x)$  and  $j(x)$

8 \_\_\_\_\_

9. The element Americium has a half-life of 25 minutes. Given an initial amount,  $A_0$ , which expression could be used to determine the amount of Americium remaining after  $t$  minutes?

(1)  $A_0 \left(\frac{1}{2}\right)^{\frac{t}{25}}$

(2)  $A_0(25)^{\frac{t}{2}}$

(3)  $25 \left(\frac{1}{2}\right)^t$

(4)  $A_0 \left(\frac{1}{2}\right)^{25t}$

9 \_\_\_\_\_

10. Which function has the greatest  $y$ -intercept?

(1)  $f(x) = 4\sin(2x)$

(3)  $h(x) = 5e^{2x} + 3$

(2)  $g(x) = 3x^4 + 2x^3 + 7$

(4)  $j(x) = 6\log_2(3x + 4)$

10 \_\_\_\_\_

11. According to the USGS, an agency within the Department of Interior of the United States, the frog population in the U.S. is decreasing at the rate of 3.79% per year. A student created a model,  $P = 12,150(0.962)^t$ , to estimate the population in a pond after  $t$  years. The student then created a model that would predict the population after  $d$  decades. This model is best represented by

(1)  $P = 12,150(0.461)^d$

(3)  $P = 12,150(0.996)^d$

(2)  $P = 12,150(0.679)^d$

(4)  $P = 12,150(0.998)^d$

11 \_\_\_\_\_



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12. What is the value of  $\tan\theta$  when  $\sin\theta = \frac{2}{5}$  and  $\theta$  is in quadrant II?

- (1)  $\frac{-\sqrt{21}}{5}$       (2)  $\frac{-\sqrt{21}}{2}$       (3)  $\frac{-2}{\sqrt{21}}$       (4)  $\frac{2}{\sqrt{21}}$       12 \_\_\_\_\_

13. A population is normally distributed with a mean of 23 and a standard deviation of 1.2. The percentage of the population that falls below 21, to the nearest hundredth, is

- (1) 0.05      (2) 4.78      (3) 8.29      (4) 91.30      13 \_\_\_\_\_

14. Audra is interested in studying the number of students entering kindergarten in the Ahlville Central School District over the next several years. Using data dating back to 2015, she determines that the number of kindergarteners is decreasing at an exponential rate. She creates a formula to model this situation  $y = a(b)^x$ , where  $x$  is the number of years since 2015 and  $y$  is the number of students entering kindergarten. If there were 105 students entering kindergarten in Ahlville in 2015, which statement about Audra's formula is true?

- (1)  $a$  is positive and  $b$  is negative.      (3) Both  $a$  and  $b$  are positive.  
 (2)  $a$  is negative and  $b$  is positive.      (4) Both  $a$  and  $b$  are negative.      14 \_\_\_\_\_

15. The solution set for the equation  $\sqrt{3(x+6)} = x$  is

- (1)  $\{6, -3\}$       (2)  $\{-6, 3\}$       (3)  $\{6\}$       (4)  $\{-3\}$       15 \_\_\_\_\_

16. The George family would like to borrow \$45,000 to purchase a new boat. They qualified for a loan with an annual interest rate of 6.75%. The monthly loan payment can be found using the formula below.

$$M = \frac{P\left(\frac{r}{12}\right)\left(1 + \frac{r}{12}\right)^n}{\left(1 + \frac{r}{12}\right)^n - 1}$$

$M$  = monthly payment

$P$  = amount borrowed

$r$  = annual interest rate

$n$  = number of monthly payments

What is the monthly payment if they would like to pay off the loan in five years?

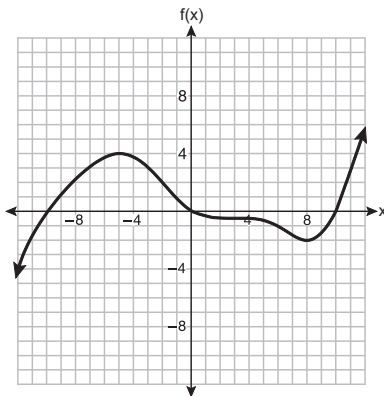
- (1) \$262.99      (2) \$252.13      (3) \$915.24      (4) \$885.76      16 \_\_\_\_\_

17. A retailer advertises that items will be discounted by 10% every Monday until they are sold. In how many weeks will an item costing \$50 first be sold for under half price?

- (1) 7      (2) 6      (3) 5      (4) 4      17 \_\_\_\_\_

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18. The graph of the function  $f(x)$  is shown.



In which interval is  $f(x)$  always positive?

- (1)  $(-2, 4)$   
 (2)  $(0, 10)$   
 (3)  $(-12, -5)$   
 (4)  $(-10, 0)$

18 \_\_\_\_\_

19. If  $f(x) = (x^2 + 3x + 2)(x^2 - 4x + 3)$  and  $g(x) = x^2 - 9$ , then how many real solutions are there to the equation  $f(x) = g(x)$ ?

- (1) 1                      (2) 2                      (3) 6                      (4) 4

19 \_\_\_\_\_

20. Which expression is a factor of  $x^4 - x^3 - 11x^2 + 5x + 30$ ?

- (1)  $x + 2$                       (2)  $x - 2$                       (3)  $x + 5$                       (4)  $x - 5$

20 \_\_\_\_\_

21. The expression  $\frac{x^2 + 6}{x^2 + 4}$  is equivalent to

- (1)  $\frac{6}{4}$     (2)  $1 + \frac{10}{x^2 + 4}$     (3)  $1 - \frac{2}{x^2 + 4}$     (4)  $1 + \frac{2}{x^2 + 4}$

21 \_\_\_\_\_

22. Stone Manufacturing has developed a cost model,  $C(x) = 0.18x^3 + 0.02x^2 + 4x + 180$ , where  $x$  is the number of sprockets sold, in thousands. The sale price can be modeled by  $S(x) = 95.4 - 6x$  and the company's revenue by  $R(x) = x \cdot S(x)$ .

The company profits,  $R(x) - C(x)$ , could be modeled by

- (1)  $0.18x^3 + 6.02x^2 + 91.4x + 180$   
 (2)  $0.18x^3 - 5.98x^2 - 91.4x + 180$   
 (3)  $-0.18x^3 - 6.02x^2 + 91.4x - 180$   
 (4)  $0.18x^3 + 5.98x^2 + 99.4x + 180$

22 \_\_\_\_\_

23. Which function is even?

- (1)  $f(x) = x^3 + 2$                       (3)  $f(x) = |x + 2|$   
 (2)  $f(x) = x^2 + 1$                       (4)  $f(x) = \sin(2x)$

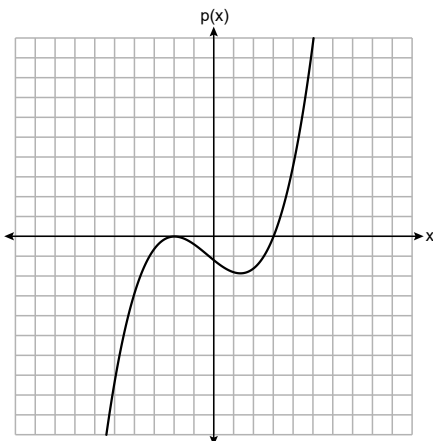
23 \_\_\_\_\_

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24. The graph of a cubic polynomial function  $p(x)$  is shown.

If  $p(x)$  is written as a product of linear factors, which factor would appear twice?

- (1)  $x - 2$                       (3)  $x - 3$   
 (2)  $x + 2$                       (4)  $x + 3$



24 \_\_\_\_\_

**Part II**

Answer all 8 questions in this part. Each correct answer will receive 2 credits. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. Utilize the information provided for each question to determine your answer. Note that diagrams are not necessarily drawn to scale. For all questions in this part, a correct numerical answer with no work shown will receive only 1 credit. All answers should be written in pen, except for graphs and drawings, which should be done in pencil. [16]

25. Factor the expression  $2x^3 - 3x^2 - 18x + 27$  completely.

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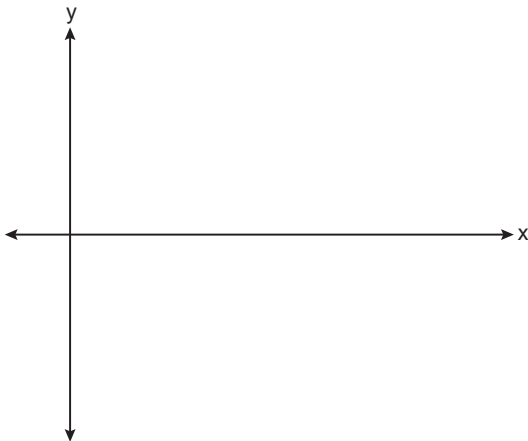
26. Algebraically determine the values of  $x$  that satisfy the system of equations shown below:

$$y = x^2 + 8x - 5$$

$$y = 8x - 4$$

27. Solve the equation  $3x^2 + 5x + 8 = 0$ . Write your solution in  $a + bi$  form.

28. On the coordinate plane below, sketch *at least one cycle* of a cosine function with a midline at  $y = -2$ , an amplitude of 3, and a period of  $\frac{\pi}{2}$ .



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29. Given  $i$  is the imaginary unit, simplify  $(5xi^3 - 4i)^2$  as a polynomial in standard form.
30. Consider the parabola given by  $y = \frac{1}{4}x^2 + x + 8$  with vertex  $(-2, 7)$  and focus  $(-2, 8)$ . Use this information to explain how to determine the equation of the directrix.

31. Write  $\frac{x\sqrt{x^3}}{\sqrt[3]{x^5}}$  as a single term in simplest form, with a rational exponent.

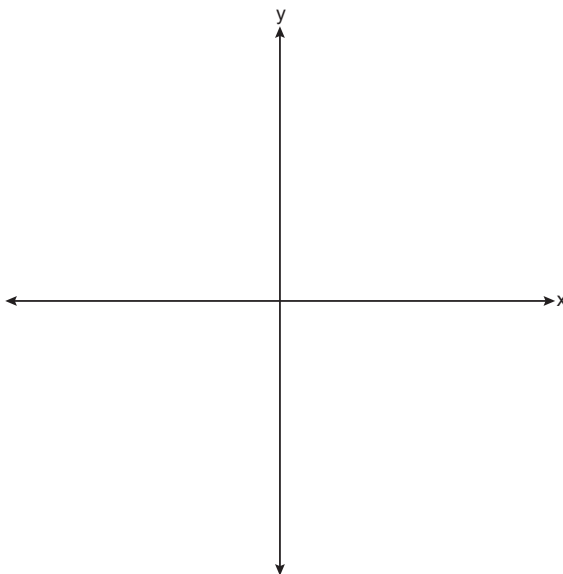
32. A fruit fly population can be modeled by the equation  $P = 10(1.27)^t$ , where  $P$  represents the number of fruit flies after  $t$  days. What is the average rate of change of the population, rounded to the *nearest hundredth*, over the interval  $[0, 10.5]$ ? Include appropriate units in your answer.

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Part III

Answer all 4 questions in this part. Each correct answer will receive 4 credits. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. Utilize the information provided for each question to determine your answer. Note that diagrams are not necessarily drawn to scale. For all questions in this part, a correct numerical answer with no work shown will receive only 1 credit. All answers should be written in pen, except for graphs and drawings, which should be done in pencil. [16]

33. Sketch  $p(x) = -\log_2(x + 3) + 2$  on the axes below



Describe the end behavior of  $p(x)$  as  $x \rightarrow -3$ .

Describe the end behavior of  $p(x)$  as  $x \rightarrow \infty$

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34. Solve for  $x$  algebraically:  $\frac{1}{x-6} + \frac{x}{x-2} = \frac{4}{x^2 - 8x + 12}$

35. Solve the following system of equations algebraically for  $x$ ,  $y$ , and  $z$ .

$$2x + 4y - 3z = 12$$

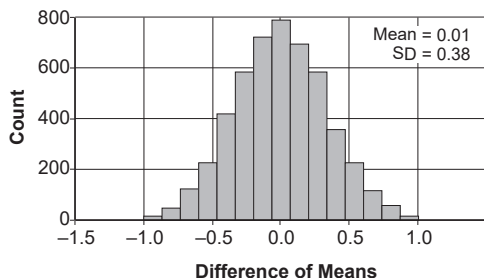
$$3x - 2y + 2z = -9$$

$$-x + y - 3z = 0$$



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36. Two classes of students were entered into an experiment to see whether using an interactive whiteboard leads to better grades. It was observed that the mean grade of students in the class with the interactive whiteboard was 0.6 points higher than the class without it. To determine if the observed difference is statistically significant, the classes were rerandomized 5000 times to study these random differences in the mean grades. The output of the simulation is summarized in the histogram below.



Determine an interval containing the middle 95% of the simulation results. Round your answer to the *nearest hundredth*.

Does the interval indicate that the difference between the classes' grades is significant? Explain.

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## Part IV

Answer the question in this part. A correct answer will receive 6 credits. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. Utilize the information provided to determine your answer. Note that diagrams are not necessarily drawn to scale. A correct numerical answer with no work shown will receive only 1 credit. All answers should be written in pen, except for graphs and drawings, which should be done in pencil. [6]

37. The Manford family started savings accounts for their twins, Abby and Brett, on the day they were born. They invested \$8000 in an account for each child. Abby's account pays 4.2% annual interest compounded quarterly. Brett's account pays 3.9% annual interest compounded continuously.

Write a function,  $A(t)$ , for Abby's account and a function,  $B(t)$ , for Brett's account that calculates the value of each account after  $t$  years.

Determine who will have more money in their account when the twins turn 18 years old, and find the difference in the amounts in the accounts to the *nearest cent*.

Algebraically determine, to the *nearest tenth of a year*, how long it takes for Brett's account to triple in value.

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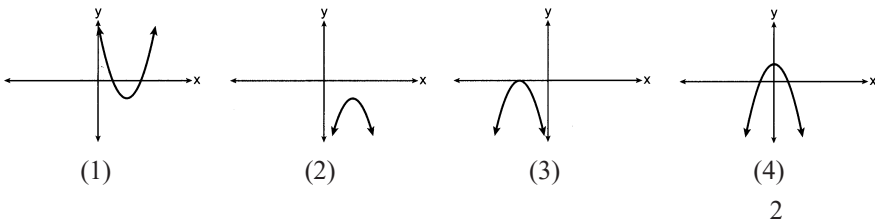
## Part I

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1. A cafeteria food manager studied the lunchtime eating habits of a group of employees in their office building. The purpose of the study was to determine the proportion of employees who purchased lunch in the cafeteria, brought their lunch from home, or purchased lunch from an outside vendor. This collection of data would best be classified as

- (1) a census (2) an experiment (3) an observational study (4) a simulation 1 \_\_\_\_\_

2. Which graph has imaginary roots?



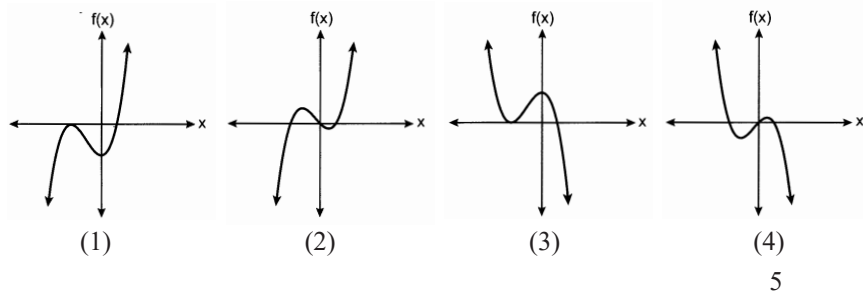
3. Given 3 is a root of  $f(x) = x^4 - x^3 - 21x^2 + 45x$ , what are the other unique roots of  $f(x)$ ?

- (1) -5, only (2) -5 and 0 (3) -3, 1 and 5 (4) -5, -3 and 0 3 \_\_\_\_\_

4. Given  $p \neq q$ ,  $p = \left(\frac{1}{2}\right)^q$ , expressed in logarithmic form, is equivalent to

- (1)  $\log_p\left(\frac{1}{2}\right) = q$  (2)  $\log_q(p) = \frac{1}{2}$  (3)  $\log_{\frac{1}{2}}(p) = q$  (4)  $\log_{\frac{1}{2}}(q) = p$  4 \_\_\_\_\_

5. Which graph best represents the graph of  $f(x) = (x + a)^2(x - b)$ , where  $a$  and  $b$  are positive real numbers?



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6. The equations  $y = 3t + 6$  and  $y = (1.82)^t$  approximately model the growth of two separate populations where  $t > 0$ . What is the best approximation of the time,  $t$ , at which the populations are the same?

- (1) -1.9                      (2) 0.3                      (3) 5.1                      (4) 21.3                      6 \_\_\_\_\_

7. Given  $y = -2x$  and  $x^2 + y^2 = 5$ , the point of intersection in Quadrant II is

- (1) (1, -2)                      (2) (-2, 1)                      (3) (-1, 1)                      (4) (-1, 2)                      7 \_\_\_\_\_

8. The rational expression  $\frac{2x^4 - 5x^2 + 3x - 2}{x - 3}$  is equivalent to

- (1)  $2x^3 - 5x - 12 - \frac{38}{x-3}$                       (3)  $2x^3 - 5x + 18 - \frac{56}{x-3}$   
 (2)  $2x^3 + 6x^2 + 13x + 42 + \frac{124}{x-3}$                       (4)  $2x^3 - 6x^2 + 13x - 36 + \frac{106}{x-3}$                       8 \_\_\_\_\_

9. The equation of the parabola that has its focus at the point  $(-3, 2)$  and directrix at  $y = 0$  is

- (1)  $y = \frac{1}{4}(x + 3)^2 + 1$                       (3)  $y = \frac{1}{8}(x + 3)^2 + 1$   
 (2)  $y = \frac{1}{4}(x - 3)^2 + 1$                       (4)  $y = \frac{1}{8}(x - 3)^2 + 1$                       9 \_\_\_\_\_

10. The seventh term of the geometric sequence

$$\sqrt{6}, -2\sqrt{3}, 2\sqrt{6}, -4\sqrt{3} \dots$$

- (1)  $6\sqrt{6}$                       (2)  $-6\sqrt{3}$                       (3)  $8\sqrt{6}$                       (4)  $-8\sqrt{3}$                       10 \_\_\_\_\_

11. A company wishes to determine the cooking time for one pound of spaghetti. The company's technicians cooked one pound of spaghetti and recorded the time needed for the spaghetti to be ready to eat. Repeating this process 35 times resulted in an approximately normal distribution, with a mean of 9.82 minutes and a standard deviation of 1.4 minutes. In which interval should the middle 95% of cooking times fall?

- (1) (8.42, 11.22)                      (3) (9.35, 10.29)  
 (2) (7.02, 12.62)                      (4) (6.82, 11.32)                      11 \_\_\_\_\_

12. Given  $f(x) = 2x^2 + 7x - 15$  and  $g(x) = 3 - 2x$ , what is  $\frac{f(x)}{g(x)}$  for all defined values?

- (1)  $-x - 5$                       (2)  $-x + 5$                       (3)  $x - 5$                       (4)  $x + 5$                       12 \_\_\_\_\_

13. Which equation is equivalent to  $P = 210x^{\frac{4}{3}}y^{\frac{7}{3}}$ ?

- (1)  $P = \sqrt[3]{210x^4y^7}$                       (3)  $P = 210xy^2\sqrt[3]{xy}$   
 (2)  $P = 70xy^2\sqrt[3]{xy}$                       (4)  $P = 210xy^2\sqrt[3]{x^3y^5}$                       13 \_\_\_\_\_



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20. Robert is buying a car that costs \$22,000. After a down payment of \$4000, he borrows the remainder from a bank, a six year loan at 6.24% annual interest rate. The following formula can be used to calculate his monthly loan payment.

$$R = \frac{(P)(i)}{1 - (1 + i)^{-t}}$$

$R$  = monthly payment

$P$  = loan amount

$i$  = monthly interest rate

$t$  = time, in months

Robert's monthly payment will be

- (1) \$298.31      (2) \$300.36      (3) \$307.35      (4) \$367.10      20 \_\_\_\_\_

21. Given  $\tan \theta = -\frac{4}{3}$  where  $\frac{\pi}{2} < \theta < \pi$ , what is the value of  $\sec \theta$ ?

- (1)  $-\frac{5}{3}$       (2)  $-\frac{3}{5}$       (3)  $\frac{4}{5}$       (4)  $\frac{5}{3}$       21 \_\_\_\_\_

22. To solve the equation  $\frac{7}{x+7} + \frac{4x}{x-7} = \frac{3x+7}{x-7}$ , Joan's first step is to multiply both sides by the least common denominator. Which statement is true?

- (1)  $-14$  is an extraneous solution.      (3)  $7$  is an extraneous solution.  
 (2)  $7$  and  $-7$  are extraneous solutions.      (4) There are no extraneous solutions.

22 \_\_\_\_\_

23. Beginning July 1, 2019, Michelle deposited \$250 into an account that yields 0.15% each month. She continued to make \$250 deposits into this account on the first of each month for 3 years. Which expression represents the amount of money that was in the account after her last deposit was made on June 1, 2022?

- (1)  $250(1.0015)^3$       (3)  $\frac{250 - 250(1.0015)^3}{1 - 1.0015}$   
 (2)  $250(1.0015)^{36}$       (4)  $\frac{250 - 250(1.0015)^{36}}{1 - 1.0015}$       23 \_\_\_\_\_

24. A study of the red tailed hawk population in a given area shows the population,  $H(t)$ , can be represented by the function  $H(t) = 50(1.19)^t$  where  $t$  represents the number of years since the study began. In terms of the monthly rate of growth, the population can be best approximated by the function

- (1)  $H(t) = 50(1.015)^{12t}$       (3)  $H(t) = 50(1.19)^{12t}$   
 (2)  $H(t) = 50(1.15)^{\frac{t}{12}}$       (4)  $H(t) = 50(1.19)^{\frac{t}{12}}$       24 \_\_\_\_\_

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## Part II

Answer all 8 questions in this part. Each correct answer will receive 2 credits. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. Utilize the information provided for each question to determine your answer. Note that diagrams are not necessarily drawn to scale. For all questions in this part, a correct numerical answer with no work shown will receive only 1 credit. All answers should be written in pen, except for graphs and drawings, which should be done in pencil. [16]

25. Factor  $x^3 + 4x^2 - 9x - 36$ , completely.

26. Determine if  $x + 4$  is a factor of  $2x^3 + 10x^2 + 4x - 16$ . Explain your answer.

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27. An initial investment of \$1000 reaches a value,  $V(t)$ , according to the model  $V(t) = 1000(1.01)^{4t}$ , where  $t$  is the time in years.

Determine the average rate of change, to the *nearest dollar per year*, of this investment from year 2 to year 7.

28. When  $\left(\frac{1}{\sqrt[3]{y^2}}\right)y^4$  is written in the form  $y^n$ , what is the value of  $n$ ? Justify your answer.

29. The heights of the members of a ski club are normally distributed. The average height is 64.7 inches with a standard deviation of 4.3 inches. Determine the percentage of club members, to the *nearest percent*, who are between 67 inches and 72 inches tall.



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30. The explicit formula  $a_n = 6 + 6n$  represents the number of seats in each row in a movie theater, where  $n$  represents the row number. Rewrite this formula in recursive form.

31. Express  $(2x^3 - 3y)^2$  in simplest form.

32. A survey was given to 1250 randomly selected high school students at the end of their junior year. The survey offered four post-graduation options: two-year college, four-year college, military, or work. Of the 1250 responses, 475 chose a four-year college. State *one* possible conclusion that can be made about the population of high school juniors, based on this survey.

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## Part III

Answer all 4 questions in this part. Each correct answer will receive 4 credits. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. Utilize the information provided for each question to determine your answer. Note that diagrams are not necessarily drawn to scale. For all questions in this part, a correct numerical answer with no work shown will receive only 1 credit. All answers should be written in pen, except for graphs and drawings, which should be done in pencil. [16]

33. A researcher wants to determine if nut allergies and milk allergies are related to each other. The researcher surveyed 1500 people and asked them if they are allergic to nuts or milk. The survey results are summarized in the table.

	Allergic to Nuts	Not Allergic to Nuts
Allergic to Milk	3	42
Not Allergic to Milk	12	1443

Determine the probability that a randomly selected survey respondent is allergic to milk.

Determine the probability that a randomly selected survey respondent is allergic to milk, given that the person is allergic to nuts.

Based on the survey data, determine whether nut allergies and milk allergies are independent events. Justify your answer.

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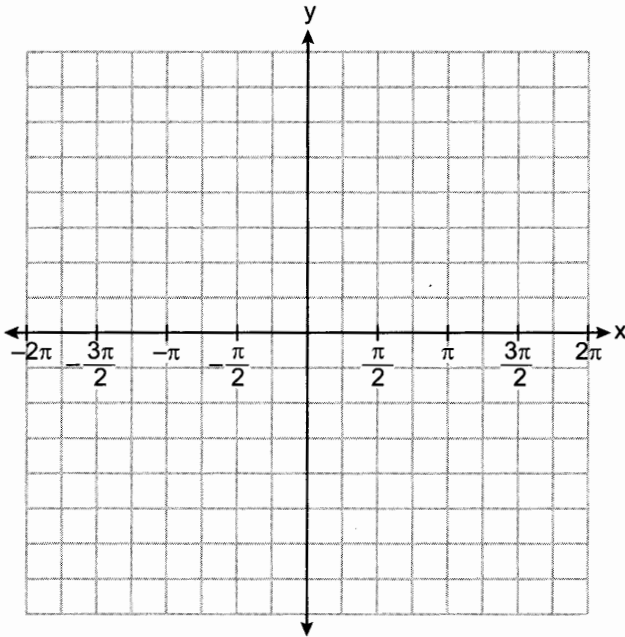
34. Algebraically solve for  $x$ :  $2x = 6 + 2\sqrt{x-1}$

35. During the summer, Adam saved \$4000 and Betty saved \$3500. Adam deposited his money in Bank  $A$  at an annual rate of 2.4% compounded monthly. Betty deposited her money in Bank  $B$  at an annual rate of 4% compounded quarterly. Write two functions that represent the value of each account after  $t$  years if no other deposits or withdrawals are made, where Adam's account value is represented by  $A(t)$ , and Betty's by  $B(t)$ .

Using technology, determine, to the *nearest tenth of a year*, how long it will take for the two accounts to have the same amount of money in them. Justify your answer.

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36. On the graph, draw *at least one* complete cycle of a sine graph passing through point  $(0, 2)$  that has an amplitude of 3, a period of  $\pi$ , and a midline at  $y = 2$ .



Based on your graph, state an interval in which the graph is increasing.

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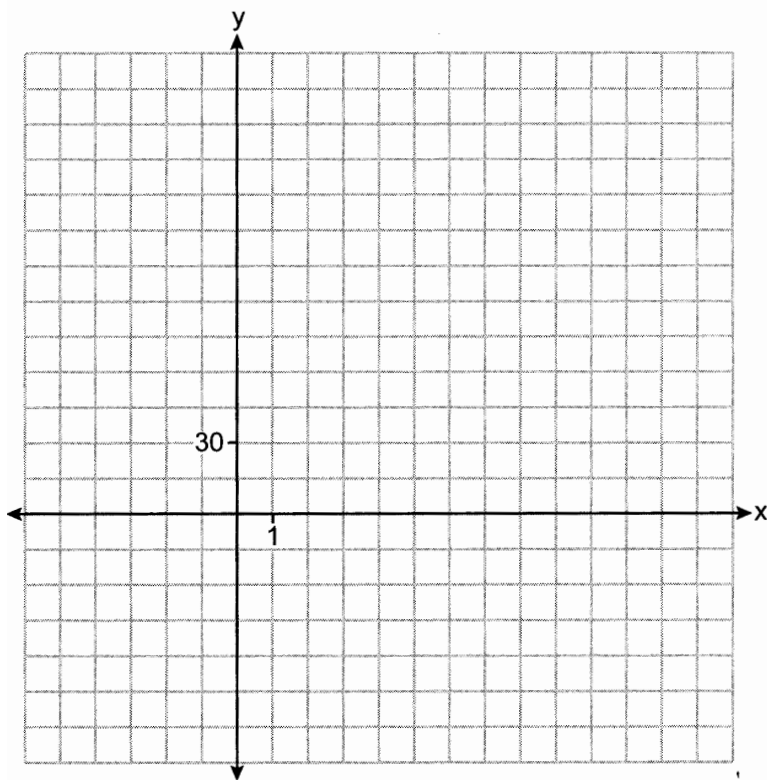
## Part IV

Answer the question in this part. A correct answer will receive 6 credits. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. Utilize the information provided to determine your answer. Note that diagrams are not necessarily drawn to scale. A correct numerical answer with no work shown will receive only 1 credit. All answers should be written in pen, except for graphs and drawings, which should be done in pencil. [6]

37. A manufacturer of sweatshirts finds that profits and costs fluctuate depending on the number of products created. Creating more products doesn't always increase profits because it requires additional costs, such as building a larger facility or hiring more workers. The manufacturer determines the profit,  $p(x)$ , in thousands of dollars, as a function of the number of sweatshirts sold,  $x$ , in thousands. This function,  $p$ , is given below.

$$p(x) = -x^3 + 11x^2 - 7x - 69$$

Graph  $y = p(x)$ , over the interval  $0 \leq x \leq 9$ , on the set of axes.



Question 37 is continued on the next page.

**Question 37 is continued**

Over the given interval, state the coordinates of the maximum of  $p$  and round all values to the *nearest integer*. Explain what this point represents in terms of the number of sweatshirts sold and profit.

Determine how many sweatshirts, to the *nearest whole sweatshirt*, the manufacturer would need to produce in order to first make a positive profit. Justify your answer.

## NOTES

