Geometry Made Easy Handbook

Next Generation Learning Standards

Introduction and Acknowledgments

The Next Generation Learning Standards (NGLS) have been developed to be straightforward. The Geometry NGLS are more direct and relevant than the predecessor. It is my hope that the students will appreciate geometry as they realize how applicable it is to everyday life. Shapes, sizes, and the relationships between them are part of our everyday thinking processes.

In preparing my handbook, I want to thank several people who were invaluable. Math is a very tough subject to type – there are so many symbols and technicalities. My thanks to my former colleague and current director of Math & Science in Hyde Park, NY, Kimberly Knisell. She helped me to interpret some of the finer points of the NGLS and to stay organized, and she also proofread. Thank you to my daughter, Debra Casey Rainha who teaches high school math at Andover High School in Massachusetts. She kept an eye on my technical notations like the bars over segments, use of *m* or not for angle measure, etc. My publisher, Keith Williams and his assistant, Julieen Sweet put everything together and Julieen makes the diagrams. That is no small feat! Keith also has a final proofreader. There is never enough proofreading in a math document.

I hope our students do well with the Next Generation Learning Standards. It is always my hope that more students will enjoy math!

Sincerely,

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Unit 1

FOUNDATIONS

- Apply geometric concepts.
- Solve using proofs.
- Recognize and use geometric terms.

1.1 Applying geometric concepts

Modeling or modeling situations are terms that are used in the Next Generation Learning Standards to describe types of problems that involve word problems, applications, real world problems, and story problems. They set up a scenario and the student is asked to solve a problem, answer a question, or to determine the best solution. Naturally there are often many ways to present a solution. Typically the student is asked to support or justify the solution. The justification or support must be done using logical reasoning and standard mathematical procedures. The justification can be a description of the procedures used (See Example 1), or it can be a written algebraic explanation (see Example 2). These are already familiar. Another type of justification or support is a geometric proof. Examples of geometric proofs are on the pages that follow.

ALGEBRAIC SOLUTIONS

Steps

1) Read the problem carefully.

2) Decide what, if any, formulas are needed. Make a diagram if possible.

3) Write an equation or substitute in the appropriate formula.

4) Solve.

5) Answer the question completely with a written conclusion.

Examples

❶ A parking garage is being constructed to contain a maximum of 100 cars. The space for each car is in the shape of a rectangle. If the average car requires a parking space that is 6 feet by 12 feet, how many square feet of parking space must be constructed? Justify your answer.

Car:

$A = lw$

$A = (6)(12)$

$A = 72\ sq\ ft$

Garage:

$100(72) = 7200\ sq\ ft$

Conclusion: Using the formula for the area of a rectangle, each car requires 72 square feet of space. Since there are a maximum of 100 cars, the garage must contain at least 7,200 square feet of parking space.

❷ Tom and Jerry are making a vegetable garden in their yard. They want to be creative, so they are designing it in the shape of a parallelogram instead of the usual rectangle or square. They want to make one side of the garden twice as long as the other. They have 24 yards of fencing to enclose the garden. What are the dimensions of the garden in feet? Would there be a difference in the measurements if the garden is rectangular instead of being a parallelogram in shape? Why or why not? Would the areas of the parallelogram shaped garden and the rectangular garden be the same? Explain.

Note: Pay attention to yards vs feet.

Let x = width of the garden in yards

$2x$ = length of the garden

Perimeter = $2l + 2w$

$24 = 2(x) + 2(2x)$

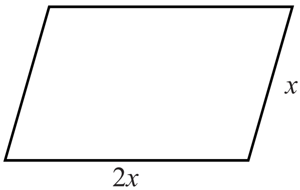
$24 = 6x$

$x = 4$ yards, $2x = 8$ yards

1 yd = 3 ft

$x = 12$ ft

$2x = 24$ ft



Conclusion: The garden has one pair of opposite sides that are 12 feet in length, and the other two sides are 24 feet in length. If the garden is made in the shape of a rectangle, the dimensions would still be 12×24 feet since the formulas for the perimeter of a rectangle and the perimeter of a parallelogram are the same. The area of the rectangular garden would be more. The rectangular garden would have an area equal to the product of the length and width. The parallelogram area would be the product of the length and the altitude of the parallelogram. The altitude is the perpendicular distance between the two lengths, and is shorter than the "width" measurement of the parallelogram. The area of the parallelogram is less than the area of the rectangular garden.

1.2 Geometric Solutions Using Proofs

A geometric proof is a specific kind of presentation of support for the multiple logical steps used to solve a geometry problem. The problem may be presented as a modeling situation, although often it is simply a problem presented directly in its mathematical form. The examples that follow are shown in mathematical form. They do not relate directly to a modeling situation, but they demonstrate the use of proofs in geometric problems. Problems presented in modeling form can be interpreted mathematically and then solved using a proof. The figures used are as they exist in a plane.

Steps

1) Read the problem. Note what is given and what needs to be proven.

2) Make a diagram and label it.

3) Consider the types of proofs and choose the format that works best for the problem. Examples are given below.

4) Start with step 1 by stating the information given.

5) Continue with logical steps leading to the statement that is to be proven. Each step must be based upon information given in the problem, or already established in the proof.

6) The final step of a proof is the statement that is to be proven.

Key Idea: Each step in a geometric proof of any kind must be based either on information given in the problem, or on steps previously completed.

TYPES OF PROOFS

- Euclidean Proof (also called Statement-Reason or Two Column Proof)
- Paragraph Proof
- Flow Proof or Flowchart
- Analytic or Coordinate Proof
- Proof by Rigid Motion
- Indirect Proof (also called Proof by Contradiction)

In the proofs throughout this handbook, abbreviations of commonly used geometric statements are used. Although these are widely accepted, not all teachers accept abbreviations. Follow your teacher's directives to receive full credit for your work.

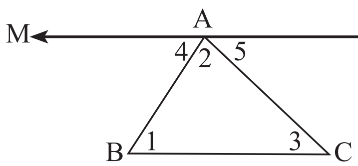
1.2

EUCLIDEAN GEOMETRY PROOF

A Euclidean Geometry Proof is a formal "statement/reason" or "2-column"proof. Each step in the progress toward the conclusion is considered to be a "statement" and is written down. Next to the statement, the mathematical reason that allowed the step to be done is written. Each step in a proof must be based on steps already completed or on given information. The last step, or statement, will be the conclusion required and next to it, the final "reason" used to get to that conclusion.

Example This problem was chosen to demonstrate the methods used to solve problems using the formal proof process. Check with your teacher for specific instructions, as there are many ways to do a proof.

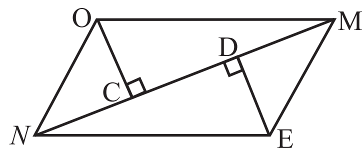
❶ **Given:** ΔABC
Prove: $m\angle 1 + m\angle 2 + m\angle 3 = 180$



Statement	Reason
1. ΔABC	1. Given.
2. Through point A, draw line $\overline{NM} \parallel \overline{BC}$ (Label angles as shown.)	2. Through a point not on a given line, there exists one and only one line parallel to the given line.
3. $\angle 1 \cong \angle 4$, $\angle 3 \cong \angle 5$	3. If two parallel lines are cut by a transversal, alternate interior angles are $\cong$.
4. $m\angle 1 = m\angle 4$; $m\angle 3 = m\angle 5$	4. Definition of $\cong$ angles.
5. $\angle 4$ and $\angle BAN$ are supplementary.	5. Two angles that form a straight line are supplementary.
6. $m\angle 4 + m\angle BAN = 180$	6. Definition of supplementary angles.
7. $m\angle BAN = m\angle 2 + m\angle 5$	7. The whole is equal to the sum of the parts.
8. $m\angle 4 + m\angle 2 + m\angle 5 = 180$	8. Substitution.
9. $m\angle 1 + m\angle 2 + m\angle 3 = 180$	9. Substitution.

❹ **Given:** Parallelogram *NEMO*, diagonal *NCDM*, $\overline{OC} \perp \overline{NM}$, $\overline{ED} \perp \overline{NM}$

Prove: $\Delta OCM \cong \Delta EDN$

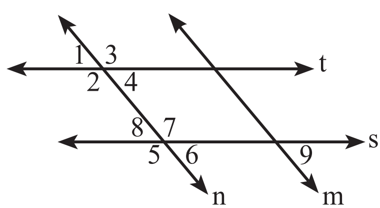


Statement	Reason
1. *NEMO* is a parallelogram $\overline{OC} \perp \overline{NM}$, $\overline{ED} \perp \overline{NM}$	1. Given.
2. $\overline{OM} \parallel \overline{NE}$, $\overline{OM} \cong \overline{NE}$	2. Opposite sides of a parallelogram are parallel and congruent.
3. $\angle OCM$ and $\angle EDN$ are right angles.	3. Perpendicular lines form right angles.
4. $\angle OCM \cong \angle EDN$	4. All right angles are congruent.
5. $\angle NMO \cong \angle MNE$	5. When two parallel lines are cut by a transversal, alternate interior angles are congruent.
6. $\Delta OCM \cong \Delta EDN$	6. AAS ≅ AAS (See page 86)

❺ **Parallel Lines Proof**

Given: $n \| m$, $t \| s$

Prove: $\angle 1 \cong \angle 9$

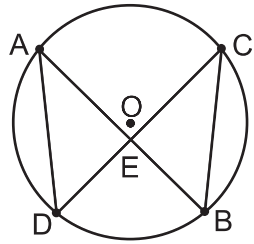


Statement	Reason
1. $n \| m$, $t \| s$	1. Given.
2. $\angle 1 \cong \angle 8$	2. If two parallel lines are cut by a transversal, corresponding angles are congruent. (*t* and *s* are the parallel lines, *n* is the transversal.)
3. $\angle 8 \cong \angle 6$	3. Vertical angles are congruent.
4. $\angle 1 \cong \angle 6$	4. Transitive Property (or Substitution).
5. $\angle 6 \cong \angle 9$	5. If two parallel lines are cut by a transversal, corresponding angles are congruent. (*n* and *m* are the parallel lines, *s* is the transversal.)
6. $\angle 1 \cong \angle 9$	6. Transitive Property (or Substitution).

PARAGRAPH PROOF

A paragraph proof is also called an "informal proof." A plan is made and the statements and reasons are written in the form of a paragraph. Included must be: the given information; what is to be proven; a description of the deductive reasoning steps; the reasons being used; a diagram when possible; and a conclusion. It could be thought of as writing a formal two column proof in a more conversational form, but all the information must be included.

Example **Paragraph Proof**



Given: Chords $\overline{AB}$ and $\overline{CD}$ of circle O intersect at E, an interior point of circle O; chords $\overline{AD}$ and $\overline{CB}$ are drawn.

Prove: $(AE)(EB) = (CE)(ED)$

We are given chords $\overline{AB}$ and $\overline{CD}$ of circle O. They intersect at E, an interior point of circle O. Chords $\overline{AD}$ and $\overline{BC}$ are drawn. $\angle A \cong \angle C$ because inscribed angles of a circle that intercept (subtend) the same arc are congruent. $\angle AED \cong \angle CEB$ because they are vertical angles. ΔAED is similar to ΔCEB because when two angles of one triangle are congruent to two angles of another triangle, the triangles are similar. In similar triangles, corresponding sides are proportional, so $\frac{AE}{CE} = \frac{ED}{EB}$. Therefore $(AE)(EB) = (CE)(ED)$ because in a proportion, the product of the means equals the product of the extremes.

1.2

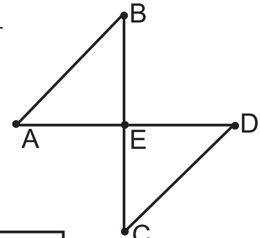
FLOW PROOF OR FLOWCHART

In a flow or chart proof, each statement is written in a box and the reason it is used is written under the box. The boxes are connected with arrows to show the sequence of the proof in reaching the conclusion. Again, the given information, what is to be proven, and a diagram are parts of this type of proof.

Examples **Flow Proofs**

❶ **Given:** E is the midpoint of $\overline{BC}$ and of $\overline{AD}$

Prove: $\Delta AEB \cong \Delta DEC$



$\boxed{E \text{ is the midpoint of } \overline{BC} \text{ and of } \overline{AD}}$

Given

↓

$\boxed{\overline{BE} \cong \overline{EC}\ ,\ \overline{AE} \cong \overline{ED}}$

Definition of midpoint.

$\boxed{\angle AEB \cong \angle DEC}$

Vertical angles are congruent.

↓

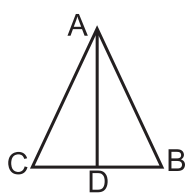
$\boxed{\Delta AEB \cong \Delta DEC}$

SAS ≅ SAS *

❷ **Given:** $\angle CAD \cong \angle BAD$

$\overline{AD} \perp \overline{CB}$

Prove: $\overline{AC} \cong \overline{AB}$



$\boxed{\overline{AD} \perp \overline{CB}}$ → $\boxed{\angle ADC \ \&\ \angle ADB \text{ are right angles}}$ → $\boxed{\angle ADC \cong \angle ADB}$

Given

Perpendicular lines from right angles.

All right angles are congruent.

$\boxed{\angle CAD \cong \angle BAD}$

Given

$\boxed{\overline{AD} \cong \overline{AD}}$

Reflexive Property

↓

$\boxed{\Delta CAD \cong \Delta BAD}$

ASA ≅ ASA *

↓

$\boxed{\overline{AC} \cong \overline{AB}}$

Corresponding parts of congruent triangles are congruent. (CPCTC)

* See page 86

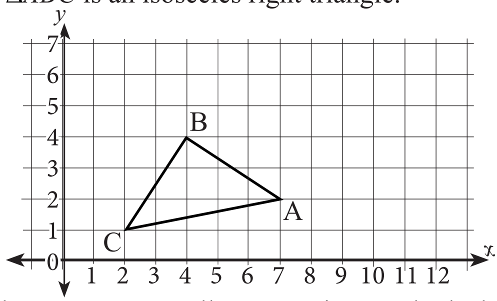
ANALYTIC OR COORDINATE PROOF

A coordinate proof is used when the problem can be represented on a coordinate plane. The points involved have (x, y) coordinates. The proof usually requires the use of the distance formula, the slope formula, the midpoint formula, or some combination of the three.

Example

Given: ΔABC with vertices $A(7, 2)$, $B(4, 4)$, and $C(2, 1)$

Prove: ΔABC is an isosceles right triangle.



Plan: There are two equally appropriate methods that can be used here.

1) Prove: $\overline{AB} \cong \overline{BC}$ and $\overline{AB} \perp \overline{BC}$.

Formulas needed are distance and slope.

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \qquad m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$d_{AB} = \sqrt{(7-4)^2 + (2-4)^2} = \sqrt{9+4} = \sqrt{13} \qquad m_{AB} = \frac{2-4}{7-4} = \frac{-2}{3} = -\frac{2}{3}$$

$$d_{BC} = \sqrt{(4-2)^2 + (4-1)^2} = \sqrt{4+9} = \sqrt{13} \qquad m_{BC} = \frac{4-1}{4-2} = \frac{3}{2}$$

Conclusion: ΔABC is a right isosceles triangle because its two legs have the same length, and they are perpendicular to each other since their slopes are negative reciprocals of each other.

2) Prove: $\overline{AB} \cong \overline{BC}$ and $AC^2 = AB^2 + BC^2$.

Formulas needed are distance and Pythagorean Theorem.

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \qquad c^2 = a^2 + b^2$$

$$d_{AB} = \sqrt{(7-4)^2 + (2-4)^2} = \sqrt{9+4} = \sqrt{13} \qquad \left(\sqrt{26}\right)^2 = \left(\sqrt{13}\right)^2 + \left(\sqrt{13}\right)^2$$

$$d_{BC} = \sqrt{(4-2)^2 + (4-1)^2} = \sqrt{4+9} = \sqrt{13} \qquad 26 = 13 + 13$$

$$d_{AC} = \sqrt{(7-2)^2 + (2-1)^2} = \sqrt{25+1} = \sqrt{26} \qquad 26 = 26$$

Conclusion: ΔABC is an isosceles triangle because its two legs are the same length. It is a right triangle because the square of the hypotenuse is equal to the sum the of the squares of the two legs.

Solve Using Proofs

FLUENCY

PROOF USING RIGID MOTION

Rigid motion transformations are reflections, rotations, and translations. If it can be shown that the figure presented in the problem shows one or more of these transformations, a rigid motion proof can be used. Any acceptable style of proof can be used. (See Unit 2 for information about rigid motions.)

PARAGRAPH PROOF USING RIGID MOTION

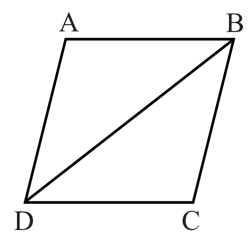
Example

Given: ΔABD is isosceles. $\Delta ABD \xrightarrow{r_{\overline{BD}}} \Delta CBD$, quadrilateral *ABCD* is formed.

Prove: *ABCD* is a rhombus.

$\overline{AB} \cong \overline{AD}$ because an isosceles triangle has two congruent sides.

A reflection is a rigid motion. Therefore distance is preserved, making $\overline{AB} \cong \overline{CB}$, $\overline{AD} \cong \overline{CD}$. Using substitution we can say that $\overline{AB} \cong \overline{CD}$, $\overline{AD} \cong \overline{CB}$. *ABCD* is a parallelogram because if a quadrilateral has both pair of opposite sides that are congruent, it is a parallelogram. Since $\overline{AB} \cong \overline{CB}$, $\overline{AD} \cong \overline{CD}$, *ABCD* is a rhombus because a parallelogram with adjacent sides that are congruent is a rhombus.



INDIRECT PROOF

Indirect proof is also called "Proof by Contradiction." This proof requires that we assume the conclusion to be drawn is false! The use of the false conclusion is called an "assumption" and is an important part of this type of proof. Through the progress of the proof, logical reasoning leads to a contradiction of the hypothesis (the "given") or some other known fact such as a theorem, definition, postulate, etc. By reaching an incorrect conclusion based on the assumption (the false original conclusion), we prove the original conclusion must be true.

An indirect proof is often used to prove that something is *not* true.

Steps

1) Write the given.

2) Using the "prove" statement, assume it is *not* true.

3) Proceed as usual trying to prove the assumption in step 2.

4) A contradiction will appear in the proof
-- a statement that is opposite something that is given or known.

5) The contradiction allows the conclusion that the original "prove" statement must be true.

Note: An indirect proof can be done using any of the acceptable proof styles.

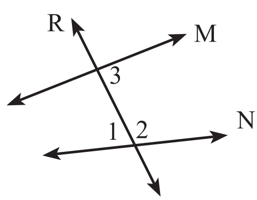
Examples

❶ **Paragraph Proof By Contradiction:**

Mary lives on Avenue M at the corner labeled 3. Nancy lives on Avenue N at the corner labeled 1. Avenue R cuts across both avenues. The city map shows that avenues M and N are NOT parallel. Nancy says the corner angles 1 and 3 are not equal. Write a geometric proof to explain.

Given: M is not $\parallel$ N

Prove: $\angle 1$ is not $\cong \angle 3$



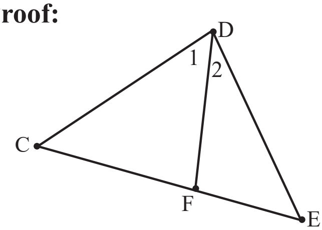
We are given that line M is not parallel to line N. Assume that $\angle 1 \cong \angle 3$. $\angle 1$ and $\angle 3$ are alternate interior angles by definition. This makes $M \parallel N$ because if two lines are cut by a transversal and alternate interior angles are congruent, the lines are parallel. This conclusion is contradictory to the given statement. Therefore if $\angle 1$ is not congruent to $\angle 3$, then M is not parallel to N.

❷ **Indirect Statement Reason Proof:**

Given: $\overline{DE} \cong \overline{DC}$

$\overline{CF}$ is not $\cong \overline{FE}$

Prove: $\angle 1$ is *not* $\cong \angle 2$



Statement	Reason
1. $\overline{DE} \cong \overline{DC}$, $\overline{CF}$ is not $\cong \overline{FE}$	1. Given.
2. $\angle 1 \cong \angle 2$	2. Assumption.
3. $\overline{DF} \cong \overline{DF}$	3. Reflexive Property.
4. $\Delta CDF \cong \Delta EDF$	4. SAS ≅ SAS (See page 86)
5. $\overline{CF} \cong \overline{FE}$	5. CPCTC *
6. $\angle 1$ is not $\cong \angle 2$	6. Contradiction of step 5 and given.

* Common abbreviation for Corresponding Parts of Congruent Triangles are Congruent.

GEOMETRIC TERMS

COMMON GEOMETRIC TERMS, DEFINITIONS, AND SYMBOLS

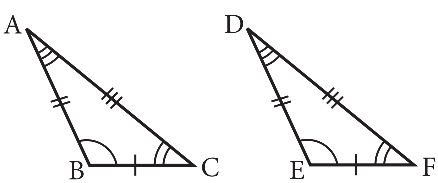
The tables that follow contain informal summaries and "short cuts" for many of the symbols, terms, and definitions that are used in plane geometry. Some can be used as reasons in a proof and others are descriptive terms. For more formal definitions, use your text or a math dictionary. Always listen to the directives of your teacher in using symbols or short cuts.

SYMBOLS USED IN PROBLEMS AND PROOFS

Description	Symbol	Example
Parallel	$\parallel$	$m \parallel n$
Perpendicular	$\perp$	$m \perp n$
Congruent	$\cong$	$\angle A \cong \angle B$
Approximately equal to	$\approx$	$\sqrt{3} \approx 1.73$
Similar	$\sim$	$\Delta ABC \sim \Delta DEF$
Maps to	$\xrightarrow[R_{45^\circ}]{}$ or $\xrightarrow{R_{45^\circ}}$	$\Delta ABC \xrightarrow{R_{45^\circ}} \Delta A'B'C'$ or $\Delta ABC \xrightarrow[R_{45^\circ}]{} \Delta A'B'C'$
Circle	$\odot$	$\odot$P

CONGRUENT VS SIMILAR

Congruent: Geometric figures are congruent if they have corresponding sides that are equal in measure and corresponding angles that are equal in measure. Rigid motion transformations result in figures that are congruent.



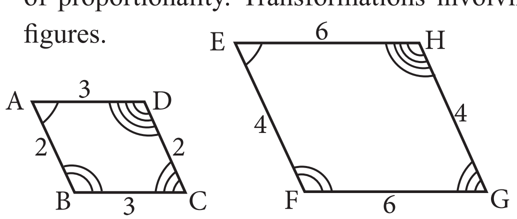
$\Delta ABC \cong \Delta DEF$

$\overline{AB} \cong \overline{DE}$ $\quad \angle A \cong \angle D$

$\overline{BC} \cong \overline{EF}$ $\quad \angle B \cong \angle E$

$\overline{CA} \cong \overline{FD}$ $\quad \angle C \cong \angle F$

Similar: Geometric figures are similar if they have corresponding angles that are equal in measure and corresponding sides that are proportional. The ratio of the proportion of the corresponding sides is called the constant of proportionality. Transformations involving dilations result in similar figures.



$\square ABCD \sim \square EFGH$

$\angle A \cong \angle E$ $\quad \dfrac{AB}{EF} = 2$ $\quad \dfrac{CD}{GH} = 2$

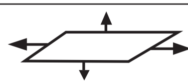
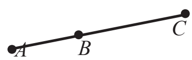
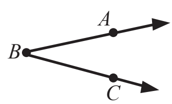
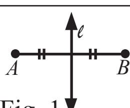
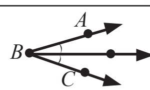
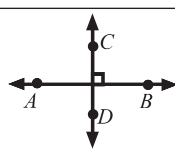
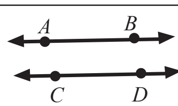
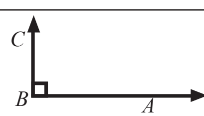
$\angle B \cong \angle F$

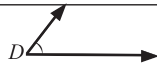
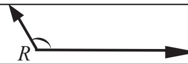
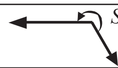
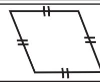
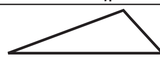
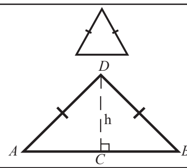
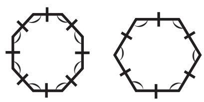
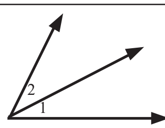
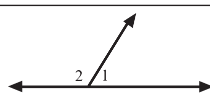
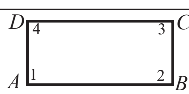
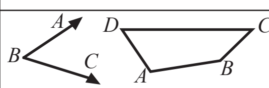
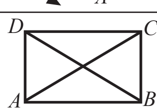
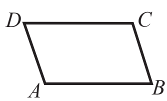
$\angle C \cong \angle G$ $\quad \dfrac{BC}{FG} = 2$ $\quad \dfrac{AD}{EH} = 2$

$\angle D \cong \angle H$

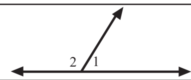
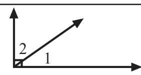
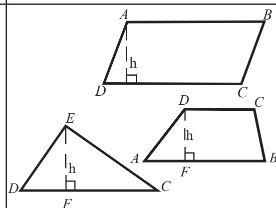
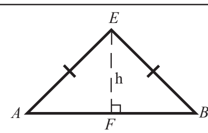
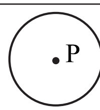
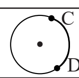
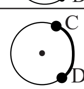
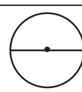
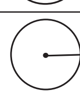
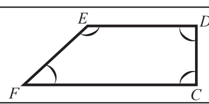
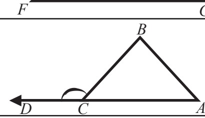
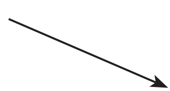
1.3

COMMONLY USED TERMS AND DEFINITIONS

Word, Term, and Mathematical Symbol	**Diagram and Labels**	**Brief Definition**
Point	• P	A location in space. Has no length, width or depth.
Line $\overleftrightarrow{AB}$		Has infinite length, no width.
Distance along a line		The length of a line between 2 points on the line.
Plane		Has infinite length and width. No depth.
Segment $\overline{AB}$		Part of line between 2 points.
Collinear Points ABC		Points that are on the same line /line segment.
Ray $\overrightarrow{CD}$		A partial line that starts at a point & goes in one direction.
Angle $\measuredangle ABC$ or $\angle ABC$		Formed when 2 rays meet at a point or when 2 lines intersect. Label with the vertex (point) at the center. The size of the opening between the rays is measured in degrees or radians.
Bisect line bisector-Figure 1 angle bisector-Fig. 2	 Fig. 1  Fig. 2	To cut in half: A bisector cuts a line segment into 2 ≅ parts, an angle into 2 ≅ angles.
Perpendicular $\overleftrightarrow{CD} \perp \overleftrightarrow{AB}$		2 lines that intersect at right angles.
Parallel $\overleftrightarrow{CD} \parallel \overleftrightarrow{AB}$		2 lines in a plane that never meet.
Right Angle		Measures 90°.

Straight Angle		Measures 180°.
Acute Angle		Measures more than 0°, less than 90°.
Obtuse Angle		Measures more than 90°, less than 180°.
Reflex Angle		Measures more than 180°, less than 360°.
Equiangular		All angles are congruent.
Equilateral		All sides are congruent.
Scalene		All sides of a figure are different lengths.
Isosceles Δ vertex angle and base angles in an isosceles Δ		2 sides are congruent: Δ*ABD* is isosceles. $\overline{AD} \cong \overline{DB}$ and $\overline{AB}$ is the base. *D* is the vertex angle, *A* and *B* are the base angles
Regular Polygon		A polygon with equal sides & equal angles. 5, 6, 8, 10 and 12 sided polygons are often used.
Adjacent Angles ∠1 and ∠2 are adjacent ∠s.		Next to each other. Angles which share only one side and a vertex but have no interior points in common.
Linear Pair		2 adjacent angles formed by the intersection of 2 lines. They are supplementary angles.
Opposite Angles ∠2 and ∠4 are opposite.		Across from each other. Not sharing a side or a vertex.
Vertex		The "point" of an angle, the corner of a polygon.
Diagonal		Connects 2 opposite vertices (corners) in a geometric figure.
Consecutive Angles /Sides ∠*A* and ∠*B* ∠*C* and ∠*D* ∠*B* and ∠*C* ∠*D* and ∠*A* $\overline{AD}$ and $\overline{DC}$ $\overline{BC}$ and $\overline{AB}$ $\overline{DC}$ and $\overline{CB}$ $\overline{AB}$ and $\overline{DA}$		Sides or angles that are "one after the other".

Geometric Terms

Pi π	π is an irrational number. The symbol π should be included in your answer if the problem says leave in terms of Pi .	The ratio of circumference to the diameter of a circle. Use the π button on your calculator to solve a problem involving π.
Supplementary Angles $m\angle 1 + m\angle 2 = 180°$		2 angles whose sum is180°. (Need not be adjacent.)
Complementary Angles $m\angle 1 + m\angle 2 = 90°$		2 angles whose sum is 90°. (Need not be adjacent.)
Base $\overline{CD}$ is the base in parallelogram *DCBA* and ΔCDE. $\overline{CD}$ and $\overline{AB}$ are both bases in trapezoid *DCBA* and *h* is the altitude.		In a formula requiring a height measurement, the base of the polygon is the side that the altitude is drawn to. The formula for the area of a trapezoid uses both bases.
Altitude and height $\overline{EF}$ is the altitude in the figure and measures the height.		Altitude $\overline{EF}$ is the segment drawn from a vertex that is ⊥ to the opposite side.
Circle ⊙P		All the points in a plane at a given distance from a given point called the center. A circle is named by the center.
Arc $\overset{\frown}{CD}$		Part of a curve between two points.
The distance of a circular arc and diameter.		The length of the circumference between two points on a circle.
Diameter		A line segment whose end-points are on the circle and passes through the center.
Radius		Distance from the center of a circle to any point on the circle (the edge).
Interior Angle of a polygon *C*, *D*, *E*, & *F* are all interior ∠'s.		An angle formed on the inside of a polygon where the sides intersect.
Exterior Angle of a polygon ∠*BCD* is exterior.		An outside angle formed by extending the side of the figure.
Vector		A directed line segment. It has magnitude represented by its length and direction shown by an arrow.

<u>Legs</u>: The two sides of the triangle that meet to form the right angle.

<u>Adjacent Leg or Side</u>: The leg of the triangle that is one side of a given angle. In figure 1, $\overline{AC}$, or leg b, is adjacent to $\angle A$. Leg a or $\overline{BC}$, is adjacent to $\angle B$. The abbreviation used in formulas is "adj.".

<u>Opposite Leg or Side</u>: The leg of the triangle that is directly across from the acute angle. It is not part of the angle itself. The leg opposite $\angle A$ is a, or $\overline{BC}$. The side opposite $\angle B$ is b, or $\overline{AC}$. The abbreviation for opposite in formulas is "opp.".

Note: In a right triangle, it is extremely important to understand the adjacent side of one acute angle is the opposite side of the other. Leg b is adjacent to $\angle A$ but is opposite to $\angle B$.

<u>Trigonometry Functions</u>

For any right triangle, certain ratios are constant for the acute angles in the triangle. Using these ratios allows us to find a missing angle or a missing side when given some information about the right triangle. Abbreviated versions of the names of the ratios are used: Sine is Sin; Cosine is Cos; and Tangent is Tan.

Sin *A* is the ratio of the *side opposite* to the given angle (A) to the *hypotenuse* of the right triangle. $\text{Sin } A = \dfrac{\text{Opp}}{\text{Hyp}}$

Cos *A* is the ratio of the *side adjacent* to the given angle (A) to the *hypotenuse* of the right triangle. $\text{Cos } A = \dfrac{\text{Adj}}{\text{Hyp}}$

Tan *A* is the ratio of the *side opposite* to the given angle (A) to the *side adjacent* to A. $\text{Tan } A = \dfrac{\text{Opp}}{\text{Adj}}$

– Many students use "SOHCAHTOA" (**S**in = **O**pp/**H**yp, **C**os = **A**dj/**H**yp, **T**an = **O**pp/**A**dj) to help remember the trig ratios.

Using angle A as the given angle:

$\text{Sin } A = \dfrac{BC}{AB}$ $\qquad$ $\text{Cos } A = \dfrac{AC}{AB}$ $\qquad$ $\text{Tan } A = \dfrac{BC}{AC}$

These ratios are also expressed as decimal numbers that match specifically to an acute angle and its associated trig function. The ratios in that form can be found in the calculator or in a table of trig values.

Examples

❶ Sin 25° = 0.422618261 in a calculator.
(In a table of trig values, it would show as 0.4226.)

❷ Tan 40° = 0.839099631 in a calculator.
(In a table of trig values, it would show as 0.8391.)

❸ If $a = 5$, $b = 12$, and c is unknown, write the *exact* ratios (NO DECIMALS) for the three trig functions of angle B.

To Solve: Sketch and label a diagram. Circle the angle involved.

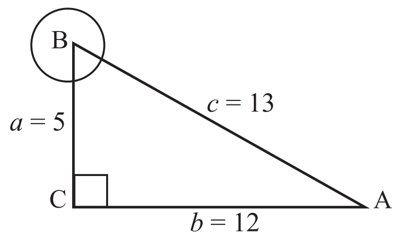
First find c.

$c^2 = a^2 + b^2$

$c^2 = 5^2 + 12^2$

$c^2 = 169$

$c = 13$



Trigonometry Functions: $Sin\,B = \dfrac{12}{13}$

$Cos\,B = \dfrac{5}{13}$

$Tan\,B = \dfrac{12}{5}$

FINDING ANGLES OR TRIG RATIO VALUES

Fortunately, we don't have to do any extended arithmetic when we need to find the value of a trig ratio. The ratios are readily available in a calculator. To find the value of the trig ratio for a particular angle, use the calculator buttons labeled "sin" or "cos" or "tan". To find the value of the angle when the trig function value is already known, use the "inverses" on the calculator. These look like "$\sin^{-1}$, $\cos^{-1}$, or $\tan^{-1}$ respectively.* Rounding of trig function values is often to the 10,000th decimal value (4th decimal place to the right of the decimal point) but in problem solving, it is best to use the full value in the calculator until the final step. Follow the directives of your teacher on the rounding procedure, as it can sometimes make a difference in the answers.

* Inverse trig functions can also be written using the prefix "arc".
The forms are equivalent.
$\text{Sin}^{-1} A = \arcsin A$
$\text{Cos}^{-1} A = \arccos A$
$\text{Tan}^{-1} A = \arctan A$

Examples

❶ $\angle A = 62°$

a) Find the value of Tan 62°. The full value that is shown in the calculator is 1.880726465. This would generally be written as Tan 62° = 1.8807.

b) Find the value of the sine of angle *A*. Sin 62° = 0.8829475929 which would round to 0.8829.

c) What is the Cos 62°? Cos 62° = 0.4694715648 ***or*** 0.4695

❷ **Cos *B* = 0.6427876097**

a) What is the measure of $\angle B$? The full value is given in this problem, so the result should be quite accurate. Use the $\cos^{-1}$ function on the calculator and type in the number that is given: $\cos^{-1}(.6427876097) = 50°$. If only the 4 digit trig value had been given, the answer might be slightly different but very close to 50°.

b) For what angle is the tangent ratio $\frac{5}{4}$?
$\text{Tan}^{-1}\left(\frac{5}{4}\right) = 51.34019175 \approx 51.34°$

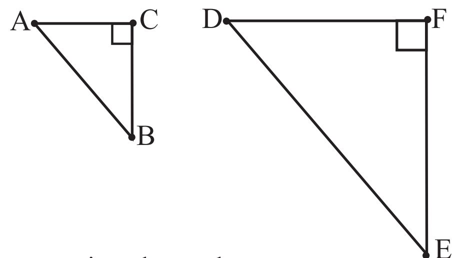
RIGHT TRIANGLES, TRIGONOMETRIC RATIOS AND SIMILARITY

Right triangle similarity allows the trig ratios to be used effectively without regard to the size of the triangle. The proportionality of the corresponding sides of similar triangles combined with the congruency of corresponding angles define the trigonometric ratios for acute angles.

Example

Given: Similar right triangles *ABC* and *DEF*. The right angles are *C* and *F*. $\angle A = \angle D$; $\angle B = \angle E$. The lengths of the sides and the hypotenuse in ΔABC and ΔDEF are in the ratio of 1:2.

Compare the trigonometric ratios of the acute angles in the two triangles.



Although the triangles are not the same size, the angles are congruent. Compare the trig ratios for the two acute angles in each triangle:

ΔABC	ΔDEF
$\text{Tan } A = \dfrac{BC}{AC}$	$\text{Tan } D = \dfrac{EF}{DF}$
$\text{Sin } A = \dfrac{BC}{AB}$	$\text{Sin } D = \dfrac{EF}{DE}$
$\text{Cos } A = \dfrac{AC}{AB}$	$\text{Cos } D = \dfrac{DF}{DE}$

Since the lengths of the sides are in a ratio of 1:2 we know that :

$$EF = 2BC \;\; ; \;\; DF = 2AC \;\; ; \;\; DE = 2AB$$

Using Substitution: $\text{Tan } D = \dfrac{2BC}{2AC}$; $\text{Sin } D = \dfrac{2BC}{2AB}$; $\text{Cos } D = \dfrac{2AC}{2AB}$

$$\text{Tan } D = \frac{2BC}{2AC} = \frac{BC}{AC} \qquad \text{Sin } D = \frac{2BC}{2AB} = \frac{BC}{AB} \qquad \text{Cos } D = \frac{2AC}{2AB} = \frac{AC}{AB}$$

Conclusion: Since the 3 trig ratios of $\angle D$ equal the 3 trig ratios of $\angle A$ respectively, using the transitive property we can say that the Tan *A* = Tan *D* ; Sin *A* = Sin *D* ; and Cos *A* = Cos *D*. So although the triangles themselves are similar triangles but different in size, it is clear that the trig ratios of the corresponding angles are equal.

The same procedure can be used to compare the trig functions of the other acute angle, *E*, and the findings will confirm the results above.

Example

Given: $\Delta ABC \sim \Delta DEF \sim \Delta STU$ with right angles C, F, and U respectively.
$a = 2$ and $b = 4$

Determine the Sin B, Cos B, and Tan B. Compare your findings with Sin E, Cos E, and Tan E. What are the three trig ratios for $\angle T$?

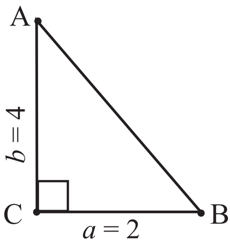


Figure 1

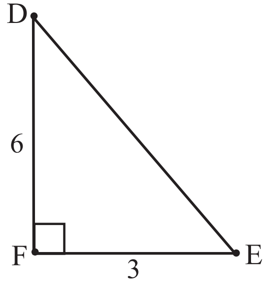


Figure 2

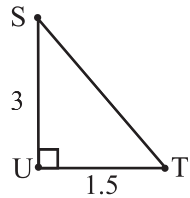


Figure 3

Solution: Both the sine and cosine functions involve the hypotenuse, so we must find the length of AB first. Use the Pythagorean Theorem. (This is where labeling with lower-case labels can be helpful. The hypotenuse in the Pythagorean Theorem is c which in ΔABC is $\overline{AB}$.) Then calculate the ratios. Find the hypotenuse of Figure 2.

In Figure 1:

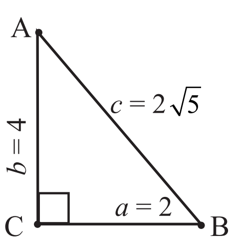
$c^2 = a^2 + b^2$ $\quad$ $\text{Sin } B = \dfrac{\text{Opp}}{\text{Hyp}} = \dfrac{AC}{AB} = \dfrac{4}{2\sqrt{5}} = \dfrac{2\sqrt{5}}{5} \approx .8944$

$c^2 = (2)^2 + (4)^2$ $\quad$ $\text{Cos } B = \dfrac{\text{Adj}}{\text{Hyp}} = \dfrac{BC}{AB} = \dfrac{2}{2\sqrt{5}} = \dfrac{\sqrt{5}}{5} \approx .4472$

$c^2 = 20$

$c = \sqrt{20} = 2\sqrt{5}$ $\quad$ $\text{Tan } B = \dfrac{\text{Opp}}{\text{Adj}} = \dfrac{AC}{BC} = \dfrac{4}{2} = 2$

Label $c = 2\sqrt{5}$



(problem continued on next page)