Geometry Made Easy Handbook

Common Core Standards Edition

By:

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Introduction

As the Common Core Standards are implemented nationally, there are new methods of teaching material that was taught in a more traditional way in the past. The new presentations associated with the standards will help our students to become "college and career ready." *Geometry Made Easy, Common Core Edition*, is meant to be a reference guide for the mathematical procedures needed to help the student complete their work in using the standards. It is not meant to be a curriculum guide and is not designed to replace any teaching methods that are used in the classroom. It is my hope that this student friendly handbook will help each geometry student to achieve success in completing his/her study of the Geometry Common Core Standards.

Sincerely,

MaryAnn Casey,
B.S. Mathematics, M.S. Education

GEOMETRY MADE EASY
Common Core Standards Edition

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Unit 1

FOUNDATIONS

- Apply geometric concepts.
- Solve using proofs.
- Recognize and use geometric terms.

1.1 Applying Geometric Concepts

Modeling or modeling situations are terms that are used in the Common Core Standards to describe types of problems that involve word problems, applications, real world problems, and story problems. They set up a scenario and the student is asked to solve a problem, answer a question, or to determine the best solution. Naturally there are often many ways to present a solution. Typically the student is asked to support or justify the solution. The justification or support must be done using logical reasoning and standard mathematical procedures. The justification can be a description of the procedures used (See Example 1), or it can be a written algebraic explanation (see Example 2). These are already familiar. Another type of justification or support is a geometric proof. Examples of geometric proofs are on the pages that follow.

ALGEBRAIC SOLUTIONS

Steps

1) Read the problem carefully.

2) Decide what, if any, formulas are needed. Make a diagram if possible.

3) Write an equation or substitute in the appropriate formula.

4) Solve.

5) Answer the question completely with a written conclusion.

Examples

❶ A parking garage is being constructed to contain a maximum of 100 cars. The space for each car is in the shape of a rectangle. If the average car requires a parking space that is 6 feet by 12 feet, how many square feet of parking space must be constructed? Justify your answer.

Car:

$A = lw$

$A = (6)(12)$

$A = 72\ sq\ ft$

Garage:

$100(72) = 7200\ sq\ ft$

Conclusion: Using the formula for the area of a rectangle, each car requires 72 square feet of space. Since there are a maximum of 100 cars, the garage must contain at least 7,200 square feet of parking space.

❷ Tom and Jerry are making a vegetable garden in their yard. They want to be creative, so they are designing it in the shape of a parallelogram instead of the usual rectangle or square. They want to make one side of the garden twice as long as the other. They have 24 yards of fencing to enclose the garden. What are the dimensions of the garden in feet? Would there be a difference in the measurements if the garden is rectangular instead of being a parallelogram in shape? Why or why not? Would the areas of the parallelogram shaped garden and the rectangular garden be the same? Explain.

Note: Pay attention to yards vs feet.

Let x = width of the garden in yards

$2x$ = length of the garden

Perimeter = $2l + 2w$

$24 = 2(x) + 2(2x)$

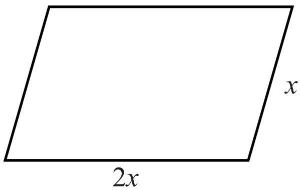
$24 = 6x$

$x = 4$ yards, $2x = 8$ yards

1 yd = 3 ft

$x = 12$ ft

$2x = 24$ ft



Conclusion: The garden has one pair of opposite sides that are 12 feet in length, and the other two sides are 24 feet in length. If the garden is made in the shape of a rectangle, the dimensions would still be 12 × 24 feet since the formulas for the perimeter of a rectangle and the perimeter of a parallelogram are the same. The area of the rectangular garden would be more. The rectangular garden would have an area equal to the product of the length and width. The parallelogram area would be the product of the length and the altitude of the parallelogram. The altitude is the perpendicular distance between the two lengths, and is shorter than the "width" measurement of the parallelogram. The area of the parallelogram is less than the area of the rectangular garden.

1.2 Geometric solutions using proofs

A geometric proof is a specific kind of presentation of support for the multiple logical steps used to solve a geometry problem. The problem may be presented as a modeling situation, although often it is simply a problem presented directly in its mathematical form. The examples that follow are shown in mathematical form. They do not relate directly to a modeling situation, but they demonstrate the use of proofs in geometric problems. Problems presented in modeling form can be interpreted mathematically and then solved using a proof.

Steps

1) Read the problem. Note what is given and what needs to be proven.

2) Make a diagram and label it.

3) Consider the types of proofs and choose the format that works best for the problem. Examples are given below.

4) Start with step 1 by stating the information given.

5) Continue with logical steps leading to the statement that is to be proven. Each step must be based upon information given in the problem, or already established in the proof.

6) The final step of a proof is the statement that is to be proven.

Key Idea: Each step in a geometric proof of any kind must be based either on information given in the problem, or on steps previously completed.

TYPES OF PROOFS

- Euclidean Proof (also called Statement-Reason or Two Column Proof)
- Paragraph Proof
- Flow Proof or Flowchart
- Analytic or Coordinate Proof
- Proof by Rigid Motion
- Indirect Proof (also called Proof by Contradiction)

In the proofs throughout this handbook, abbreviations of commonly used geometric statements are used. Although these are widely accepted, not all teachers accept abbreviations. Follow your teacher's directives to receive full credit for your work.

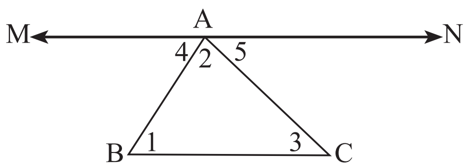
1.2

EUCLIDEAN GEOMETRY PROOF

A Euclidean Geometry Proof is a formal "statement/reason" or "2-column"proof. Each step in the progress toward the conclusion is considered to be a "statement" and is written down. Next to the statement, the mathematical reason that allowed the step to be done is written. <u>Each step in a proof must be based on steps already completed or on given information</u>. The last step, or statement, will be the conclusion required and next to it, the final "reason" used to get to that conclusion.

Example This problem was chosen to demonstrate the methods used to solve problems using the formal proof process. Check with your teacher for specific instructions, as there are many ways to do a proof.

❶ **Given:** ΔABC
Prove: $m\angle 1 + m\angle 2 + m\angle 3 = 180$

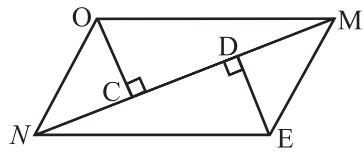


Statement	Reason
1. ΔABC	1. Given.
2. Through point A, draw line $\overline{NM} \parallel \overline{BC}$ (Label angles as shown.)	2. Through a point not on a given line, there exists one and only one line parallel to the given line.
3. $\angle 1 \cong \angle 4$, $\angle 3 \cong \angle 5$	3. If two parallel lines are cut by a transversal, alternate interior angles are $\cong$.
4. $m\angle 1 = m\angle 4$; $m\angle 3 = m\angle 5$	4. Definition of $\cong$ angles.
5. $\angle 4$ and $\angle BAN$ are supplementary.	5. Two angles that form a straight line are supplementary.
6. $m\angle 4 + m\angle BAN = 180$	6. Definition of supplementary angles.
7. $m\angle BAN = m\angle 2 + m\angle 5$	7. The whole is equal to the sum of the parts.
8. $m\angle 4 + m\angle 2 + m\angle 5 = 180$	8. Substitution.
9. $m\angle 1 + m\angle 2 + m\angle 3 = 180$	9. Substitution.

1.2

❹ **Given:** Parallelogram *NEMO*, diagonal *NCDM*, $\overline{OC} \perp \overline{NM}$, $\overline{ED} \perp \overline{NM}$

Prove: $\Delta OCM \cong \Delta EDN$

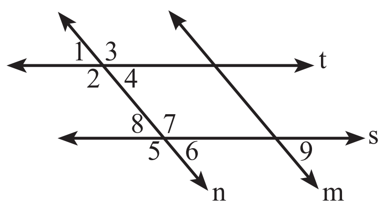


Statement	Reason
1. *NEMO* is a parallelogram $\overline{OC} \perp \overline{NM}$, $\overline{ED} \perp \overline{NM}$	1. Given.
2. $\overline{OM} \parallel \overline{NE}$, $\overline{OM} \cong \overline{NE}$	2. Opposite sides of a parallelogram are parallel and congruent.
3. $\angle OCM$ and $\angle EDN$ are right angles.	3. Perpendicular lines form right angles.
4. $\angle OCM \cong \angle EDN$	4. All right angles are congruent.
5. $\angle NMO \cong \angle MNE$	5. When two parallel lines are cut by a transversal, alternate interior angles are congruent.
6. $\Delta OCM \cong \Delta EDN$	6. AAS ≅ AAS

❺ **Parallel Lines Proof**

Given: $n \parallel m$, $t \parallel s$

Prove: $\angle 1 \cong \angle 9$



Statement	Reason
1. $n \parallel m$, $t \parallel s$	1. Given.
2. $\angle 1 \cong \angle 8$	2. If two parallel lines are cut by a transversal, corresponding angles are congruent. (*t* and *s* are the parallel lines, *n* is the transversal.)
3. $\angle 8 \cong \angle 6$	3. Vertical angles are congruent.
4. $\angle 1 \cong \angle 6$	4. Transitive Property (or Substitution).
5. $\angle 6 \cong \angle 9$	5. If two parallel lines are cut by a transversal, corresponding angles are congruent. (*n* and *m* are the parallel lines, *s* is the transversal.)
6. $\angle 1 \cong \angle 9$	6. Transitive Property (or Substitution).

Unit 4

Extending to Three Dimensions

- Explain volume formulas and use them to solve problems.
- Visualize relationships between two-dimensional and three-dimensional objects.
- Apply geometric concepts in modeling situations.

VOLUME

Cylinder: The volume of a prism is, in general, the area of the base multiplied by the height of the prism. In the case of a cylinder, the base is a circle. Since the area of a circle is known to be $A = \pi r^2$, as is demonstrated on page 142, this area formula needs to be multiplied by the perpendicular distance between the bases of the cylinder, called the altitude or the height.

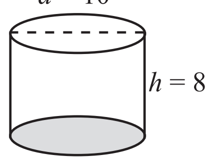
Example What is the volume, in cubic inches, of a can of coffee that has a diameter of 10 inches and is 8 inches tall? Remember to use the radius of 5 in the circle area formula.

$V = Bh$

$V = \pi r^2 h$

$V = \pi(5)^2 8$

$V = 200\pi$



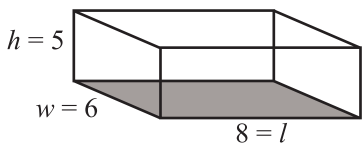
Rectangular Solid: Since the faces of a rectangular solid are all rectangles, choosing one face as the base, finding its area using $A = lw$, then multiplying by the length of the side perpendicular to it will produce the area of the rectangular prism. $A = lwh$.

Example

$V = lwh$

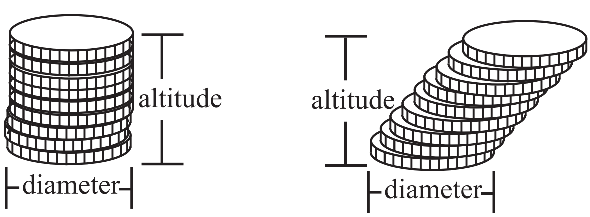
$V = (5)(6)(8)$

$V = 240$ cubic units.



Cavalieri's Principle: This is another way to consider the volume of cylinders, prisms, and other 3 dimensional figures.

If two figures have equal altitudes and equal areas for each parallel plane that can be cut through them, their volumes are equal. Think of a stack of coins that is in a straight pile. Each coin represents a parallel plane. Then tilt the stack to one side. Each coin in the tilted pile has the same area as it counterpart in the straight pile, and both stacks have the same altitude. Cavalieri's Principle tells us that the volume for either stack of coins is the product of the base area and the altitude and that the two stacks have equal volume.



Geometric Measurement

Cones and Pyramids: These are related to cylinders and rectangular solids.

To informally demonstrate the relationship between cones and cylinders or pyramids and rectangular solids, it is best to do a physical demonstration of pouring rice (or water) from a cone into a cylinder with the same base and height. It will take three cones full of the rice to fill the cylinder. Using a pyramid with a rectangular base and doing the same procedure to fill a rectangular solid with the same base and height requires three pyramids filled with rice to fill the rectangular solid. The formulas for the volumes of the cone and pyramid are each $\frac{1}{3}$ of their associated solid.

Cone: $V = \frac{1}{3}Bh$ where $B = \pi r^2$ **Pyramid:** $V = \frac{1}{3}Bh$ where $B = lw$

The formula for the volume of the pyramid can be fairly easily demonstrated further using a cube. The cube can be "sliced" into three congruent pyramids. They have congruent bases and altitudes. Since they are congruent, they have equal volumes. That makes each one equal to $\frac{1}{3}$ the volume of the cube.

Examples Cones and Pyramids

❶ **Cone:** The radius of the base of a cone is 9. The height or altitude of the cone is 10. What is the volume of the cone?

$A_{circular\ base} = \pi r^2$

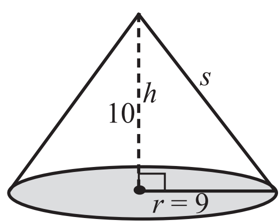
$A = (9)(9)\pi$

$V = \frac{1}{3}Bh$

$V_{cone} = \frac{1}{3}(81\pi)(10)$

$V = 270\pi$

$V_{cone} = 848.23$ cubic units



❷ **Pyramid:**

$A_{Base} = s^2$

$A_{Base} = (10)^2 = 100$

To find h, use the Pythagorean Theorem.

Note: 5-12-13 is a Pythagorean Triple

$13^2 = s^2 + h^2$

$169 = 25 + h^2$

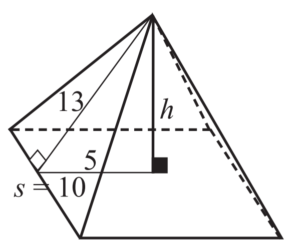
$\sqrt{144} = h$

$h = 12$

$V_{pyramid} = \frac{1}{3}Bh$*

$V = \frac{1}{3}(100)(12)$

$V_{pyramid} = 400$ cubic units



* Since B equals s^2, this formula could be written $V = s^2h$

SPHERES

Sphere: A sphere is developed by rotating a circle about any of its diameters. The formula for the volume of a sphere is $V = \frac{4}{3}\pi r^3$.

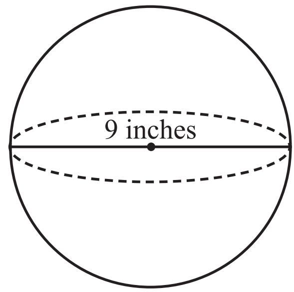
Examples

❶ What is the volume, to the *nearest tenth*, of a sphere with a diameter of 9 inches?

$V = \frac{4}{3}\pi r^3$

$V = \frac{4}{3}\pi (4.5)^3$

$V \approx 381.7$ *cubic inches*



❷ Andrew has a box shaped like a cube that contains sand. He is going to transfer the sand to spherical containers for a science project. The box is 14 inches on each side. The containers are spheres with the diameter equal to 8 inches. How many spheres will he be able to fill completely?

Solution: Find the volume of the cube and the volume one of the spherical containers will hold. Divide the total amount of sand by the volume for each sphere.

Cube:

$V = s^3$

$V = (14)^3$

$V = 2744$ *cubic inches of sand*

Sphere:

$V = \frac{4}{3}\pi r^3$

$V = \frac{4}{3}\pi (4)^3$

$V \approx 268.08$ *cubic inches*

Number of spherical containers: $\frac{V_{cube}}{V_{sphere}} = \frac{2744}{268.08} = 10.24$

Conclusion: Andrew will be able to fill 10 spherical containers completely.

Geometric Measurement

4.2

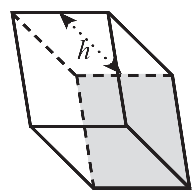
3-DIMENSIONAL FIGURES AND THEIR PROPERTIES

Solid: A closed figure, usually including its interior space.

Polyhedron: A closed figure with faces that are planar (flat). The faces meet on line segments called **edges** and the edges meet at points called **vertices**.

Cross Section: A cross section is a 2-dimensional figure that is created when a plane is passed through a polyhedron. The cross section can be many different shapes, depending on the polyhedron involved. Cross sections of solids related to circles are also created when a plane passes through a cylinder or sphere.

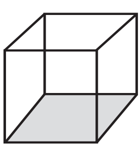
Parallelepiped: A polyhedron that has parallel bases and all the lateral faces are parallelograms. The volume of a parallelepiped equals the product of the area of the base (B) and the altitude (h) which is the perpendicular distance between the two parallel bases.



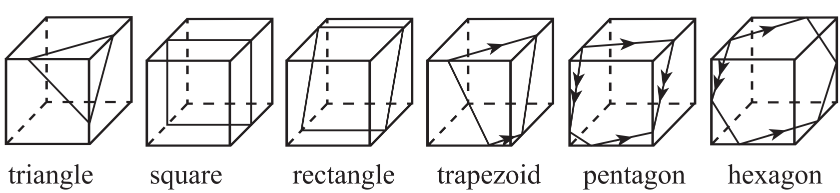
Some Parallelepipeds have specific characteristics that classify them as prisms.

Prism: A polyhedron with parallel congruent bases which are both polygons and lateral faces that are all parallelograms. The lateral edges of a prism are congruent and parallel. The height of a prism is the perpendicular distance between the parallel bases.

Cube: A prism whose faces are all congruent squares. (Ex: A die)



Cross sections of a cube:



Geometric Measurement